Origin of the Metallicity Dependence of Exoplanet Host Stars in the Protoplanetary Disk Mass Distribution

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ABSTRACT

The probability of a star hosting a planet that is detectable in radial velocity surveys increases \( P_{\text{pl}}(Z) \propto (10^Z)^2 \), where \( Z \) is stellar metallicity. Models of planet formation by core accretion reproduce this trend, since the protoplanetary disk of a high metallicity star has a high density of solids and so forms planetary cores which accrete gas before the primordial gas disk dissipates. This paper considers the origin of the form of the metallicity dependence of \( P_{\text{pl}}(Z) \). We introduce a simple model in which detectable planets form when the mass of solid material in the protoplanetary disk, \( M_s \), exceeds a critical value. In this model the form of \( P_{\text{pl}}(Z) \) is a direct reflection of the distribution of protoplanetary disk masses, \( M_g \), and the observed metallicity relation is reproduced if \( P(M_g > M_g^0) \propto (M_g^0)^{-2} \). We argue that a protoplanetary disk’s dust mass measured in sub-mm observations is a relatively pristine indicator of the mass available for planet-building and find that the disk mass distribution derived from such observations is consistent with the observed \( P_{\text{pl}}(Z) \) if a solid mass \( M_s > 0.5M_J \) is required to form detectable planets. Any planet formation model which imposes a critical solid mass for detectable planets to form would reproduce the observed metallicity relation, and core accretion models are empirically consistent with such a threshold criterion. While the outcome of planet formation in individual systems is debatable, we identify 7 protoplanetary disks which, by rigid application of this criterion, would be expected to form detectable planets and may provide insight into the physical conditions required to form such planets. A testable prediction of the model is that the metallicity dependence should flatten both for \( Z > 0.5 \) dex and as more distant and lower mass planets are discovered. Further, combining this model with one in which the evolution of a star’s debris disk is also influenced by the solid mass in its protoplanetary disk, results in the prediction that debris disks detected around stars with planets should be more infrared luminous than those around stars without planets in tentative agreement with recent observations.

Key words: circumstellar matter – stars: planetary systems: formation – stars: planetary systems: protoplanetary discs – stars: pre-main-sequence.

1 INTRODUCTION

The study of how planetary systems form and evolve was revolutionised when the first extrasolar planet was discovered in radial velocity studies of the star 51 Peg (Mayor \& Queloz 1995). Over 200 extrasolar planets are now known (Butler et al. 2006), and studying these planets has yielded enormous advances in our understanding of how they formed (Papaloizou \& Terquem 2006; Udry et al. 2007). Perhaps the most telling discovery was that of a correlation in the probability of a star hosting a planet, \( P_{\text{pl}} \), which is found to increase with stellar metallicity (Gonzalez 1997). Fischer \& Valenti (2005; hereafter FV05) found that, for stars with a metallicity \( Z = [\text{Fe}/\text{H}] \) between -0.5 and 0.5 dex, the metallicity dependence of the fraction of stars with planets with orbital periods < 4 years and with amplitudes in radial velocity studies in excess of \( K > 30 \) m s\(^{-1}\) (i.e., Saturn-Jupiter mass planets, depending on orbital period) is

\[
P_{\text{pl}}(Z) = 0.03 \times 10^{2Z},
\]

which corresponds to a planet fraction which increases with the square of the number of iron atoms in the stellar atmosphere. Similar trends have been found to apply to all species including Si and Ni (e.g., Ecuvillon et al. 2004; Robinson et

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al. 2006; Gonzalez 2006). The origin of this metallicity dependence is thought to be intrinsic to the planet formation process (FV05), and not caused by contamination from planetesimals falling onto the star, as is believed to be the cause of the high metallicities of DAZ white dwarfs (Jura 2006; Kilic & Redfield 2007), although the recent discovery that planet hosting giant stars do not favour metal rich systems is currently reigniting this debate (Pasquini et al. 2007).

Since the discovery of the extrasolar planet metallicity correlation, much work has gone into considering how stellar metallicity could affect different aspects of the planet formation process in the various models (e.g., Livio & Pringle 2003). It has been found that forming planets by gravitational instability does not introduce any significant metallicity dependence (Boss 2002; Cai et al. 2006), whereas models of planet formation by core accretion seem to readily reproduce the observed trend (Ida & Lin 2004b; Kornet et al. 2005; Benz et al. 2006; Robinson et al. 2006). This is because, in the core accretion models, planetesimals grow into planet cores through collisions, subsequently accreting gas from the surrounding gas disk once they become large enough, and then interacting with that disk so as to migrate inward (e.g., Lin & Papaloizou 1986; Papaloizou et al. 2007).

The core accretion models predict a metallicity dependence because a higher metallicity implies higher solid mass and hence faster core growth, which means that the critical core mass for gas accretion can occur before the gas disk dissipates on ~ 6 Myr timescales (Haisch, Lada & Lada 2001; Clarke, Gendrin & Sotomayer 2001). However, it remains to be explained why the metallicity dependence has a form $\propto 10^{0.2 Z}$ as opposed to, e.g., $\propto 10^{Z}$. The origin of the dependence found in these models is hidden somewhere within the large number of model components of which they are comprised, although it has been shown that a large solid mass is required if planets are to form (Ida & Lin 2004b).

In this paper we consider the origin of the form of the metallicity dependence using a simple heuristic model in which detectable planets form as long as the solid mass of the planet cores through collisions, subsequently accreting gas from the surrounding gas disk once they become large enough, and then interacting with that disk so as to migrate inward (e.g., Lin & Papaloizou 1986; Papaloizou et al. 2007). The core accretion models predict a metallicity dependence because a higher metallicity implies higher solid mass and hence faster core growth, which means that the critical core mass for gas accretion can occur before the gas disk dissipates on ~ 6 Myr timescales (Haisch, Lada & Lada 2001; Clarke, Gendrin & Sotomayer 2001). However, it remains to be explained why the metallicity dependence has a form $\propto 10^{0.2 Z}$ as opposed to, e.g., $\propto 10^{Z}$. The origin of the dependence found in these models is hidden somewhere within the large number of model components of which they are comprised, although it has been shown that a large solid mass is required if planets are to form (Ida & Lin 2004b).

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ends up frozen onto dust grains or photo-dissociated (Dullemond et al. 2007; Najita et al. 2007). On the other hand, the dust mass distribution of protoplanetary disks is well characterised, since this can be measured with relatively few uncertainties from sub-mm and mm wavelength observations (André & Montmerle 1994; Beckwith, Henning, & Nakagawa 2000).

Here we make the assumption that dust mass can be used as a proxy for the total gas mass in protoplanetary disks (for a fixed Z), and we derive the gas mass distribution from the dust mass distribution in Taurus-Auriga, which was measured using sub-mm photometry of 153 presubmain sequence stars by Andrews & Williams (2005; hereafter AW05). Since the stars in the AW05 sample are at a range of evolutionary stages we chose to use only the disk masses of the 75 class II objects (i.e., T Tauri stars) in their sample to ensure that the disk mass distribution is indicative of that at the epoch of planet formation. Class I sources were omitted because of a potential contribution to the sub-mm flux from a remnant circumstellar envelope. Class III sources were omitted because of the possibility that their currently low disk masses are a consequence of the disks being at an advanced evolutionary stage, and so are not necessarily indicative of a low mass present at the planet forming epoch.

To obtain the gas mass distribution, the gas/dust ratio was assumed to be 100 for all stars, based on the metallicities in nearby star forming regions being close to solar with a small dispersion for each region (Padgett 1996; Vuong et al. 2003; James et al. 2006). The mass distribution of class II objects is shown in Fig. 1. Ten objects from this sample have only upper limits to their disk masses, which were set to zero in Fig. 1. Since these upper limits are ≤ 1MJ, we infer that the disk mass distribution is accurate for the most massive 69% (52/75) of disks that are above this limit (≥ 1MJ).

The critical solid mass model (§2.1) was used to determine the metallicity relation predicted from the observed gas mass distribution:

\[ P_{pl} = \frac{N(M_g > M'_g)}{N_{tot}} \]  

where Poisson counting statistics were used to determine the uncertainty in the number of disks larger than a given limit in the distribution and \( N_{tot} = 75 \). The probability determined from equation (5) could be assigned a corresponding metallicity, \( Z' \), from the relation \( M_g = M_{g, crit} \). Equation (3) means that

\[ Z' = -\log 0.01M'_g/M_{g, crit}. \]  

The value of \( M_{g, crit} \) was constrained to achieve a mean planet probability for the metallicity range \( Z = 0.25 - 0.5 \) dex in agreement with that found by FV05, i.e., \( P_{pl} = 14.8 \pm 3.5\% \), giving \( M_{g, crit} = 0.5 \text{MJ} \).

\[ M_{g, crit} = (0.5 \pm 0.1)M_J. \]  

The extrasolar planet-metallicity relation predicted by this model is plotted in Fig. 2, and shows good agreement with the observed relation (equation 1). We have also inverted the problem by deducing the required disk mass distribution that would lead to the solid line in Figure 2 (i.e., \( P_{pl}(Z) \) parameterised according to

\[ P_{pl} = 0.3 \times 10^{-2}. \]
In Figure 1 we compare this required distribution with the observed gas mass distribution. Noting that this comparison can only be made over the upper quartile of disk masses (since current planet detection statistics only extend to metallicities $<0.5$ and, in the model, it is only this range of disk masses which can form planets in this metallicity regime), it is evident that there is also good agreement between the model and observed distributions when plotted in this way. To quantify this, we performed a one-sided Kolmogorov-Smirnov test to compare the distribution of gas masses inferred from AW05, when converted into metallicity (equation 6), with that inferred from equation (1) for the range $Z' = -0.5$ to 0.5 dex.

We found that discrepancies as large as or greater than those observed occur in 69% of samples of 75 members drawn from a population with a cumulative distribution function in which $P(Z < Z') = 0.03 \times 10^{0.2Z'}$; i.e., we conclude that the gas mass data are not unlikely to be drawn from such a distribution, since at least 2 out of 3 times one would expect data at least as discrepant as observed.

3 DISCUSSION

We have shown, under the assumption that a critical solid mass in the protoplanetary disk is required to form a planet that is detectable in radial velocity surveys, that the observed frequency of planet detections as a function of metallicity, $P_\text{det}(Z)$, is compatible with the observed disk mass distribution (as derived from sub-mm dust mass measurements of Classical T Tauri stars in local star forming regions). We now discuss the physical basis for this simple model and further observational tests.

3.1 Comparison with core accretion models

To consider the physical basis for the outcome of planet formation being determined solely by dust mass, we appeal to the core accretion models of Ida & Lin (2004a, 2004b; hereafter IL04). The IL04 models are local, in the sense that planet formation depends on local quantities such as gas and solid surface density. Therefore we expect any threshold effect to involve surface density rather than mass. We first assess whether the results of IL04 are compatible with the hypothesis that planet formation requires a critical metallicity independent solid surface density and return to a discussion of the relationship between solid surface density normalisation and dust mass in §3.2. We can assess this hypothesis in two ways. Firstly, we can simply take the distribution of disk surface densities assumed by IL04 (a log-normal distribution of width 1.0 dex that is centred on the surface density of the minimum mass solar nebula and truncated at $>1.48\sigma$), apply a threshold solid surface density for planet formation that is independent of metallicity and see whether we can reproduce their numerical results. Figure 3 shows that this is indeed the case: the nominal model from IL04b is well reproduced by assuming a critical solid surface density of 8 times the minimum mass solar nebula, whereas their variant models where the rate of core accretion is enhanced or reduced by a factor of three are well reproduced by models in which the critical solid surface density is respectively 4 and 22 times the minimum mass solar nebula. We stress that the IL04 models contain a large number of ingredients and do not explicitly impose a threshold criterion. Nevertheless, we see that their results are empirically equivalent to the imposition of a simple threshold.

In a second approach, we can now attempt to understand why the IL04 models behave in this way. Examination of these models shows that the formation of gas giant planets hinges on rocky cores being able to grow to a critical mass (a few $M_\oplus$) before the gas disk is dispersed. The requirement of sufficiently rapid core growth implies that they have to form inside a critical radius, $a_{\text{ig}}$, which depends on both gas and solid surface densities. On the other hand, inward of a second critical radius, $a_{\text{tg}}$ (which depends on solid surface density), a critical core mass is not achievable because the required core mass exceeds the local isolation mass (at which point the core has consumed all the material in its local feeding zone). Evidently, the formation of gas giant planets is possible only for the case $a_{\text{ig}} < a_{\text{tg}}$ and we can derive a condition on the gas and solid surface densities corresponding to the critical case where $a_{\text{ig}} = a_{\text{tg}}$. This translates into a condition on the minimum surface density of solids as a function of metallicity. We find that the critical surface density of solids scales as $10^{-0.06Z}$ (assuming, as in IL04, that a disk’s surface density scales $\Sigma \propto r^{-p}$ where $r$ is radius and $p = 1.5$). This very weak dependence on metallicity results from the fact that the growth rate of solid cores is much more strongly dependent on orbital radius than on the gas column density and hence $a_{\text{tg}}$ is only very weakly dependent on gas column density. Therefore, the threshold criterion $a_{\text{ig}} = a_{\text{tg}}$ is nearly independent of gas column density and thus the dependence of critical dust column on metallicity is extremely weak. It is this extremely weak dependence of the critical solid surface
density on metallicity which we believe to account for the
excellent correspondence between the numerical results of
IL04 and the application of our simple threshold hypothesis
(see Fig. 3).

A further test of this hypothesis would be to exam-
ine how $P_{pl}(Z)$ predicted by the core accretion models de-

dpends on the assumed distribution of disk surface densities,
since if the outcome is governed by a critical surface density
of solids for planet formation then using a narrower dis-

tribution of disk surface densities as input would result in
a steeper metallicity dependence (since in the critical solid
surface density model the metallicity dependence simply re-

dflects the disk surface density distribution used as input). In
contrast to IL04, Robinson et al. (2006) did vary this quan-
tity and indeed found that $P_{pl}(Z)$ rose more gently when a
larger range of disk surface densities was employed.

3.2 Why sub-mm dust mass determines outcome

Regardless of the comparison with core accretion models, it
is notable that the critical solid mass model fits the planet-
metallcity relation found in nature. It is, however, surprising
that sub-mm dust mass should be such a good indicator of
whether planets are going to form in a disk, since sub-mm
measurements probe the current mass in mm- to cm-sized
dust and so are not necessarily representative of the primor-
dial inventory of solid or gas mass. Indeed, class II objects in
Taurus-Auriga have a range of ages and so we would expect
the oldest stars to have already lost a significant quantity of
gas through accretion onto the star (Clarke et al. 2001). We
may also expect some loss of detectable dust mass with age
through grain growth and accretion onto the star with the
gas. However, there is no evidence that sub-mm dust mass
changes with age on the pre-main sequence (e.g., Wyatt et
al. 2003) suggesting that the mass in mm- to cm-sizes is
constant. This is to be expected, since the total dust mass
$M_{\text{dust}} \propto r_{\text{out}}^{-2p}$, where $r_{\text{out}}$ is the disk outer edge, so that as
small as $p < 2$ the sub-mm dust mass is concentrated in the
outer regions of the disk. Since typically observed values for
protoplanetary disks are $p \approx 0.85$ and $r_{\text{out}} \approx 200$ AU (An-
drews & Williams 2007), the timescale for grains containing
most of the disk mass to grow to larger than 1 metre, and
so become invisible in the sub-mm, may be expected to be
longer than the 10Myr period over which planet formation
(in the inner regions) must take place (e.g., Dullemond &
Dominik 2005). Indeed some disks cannot harbour signifi-
cant quantities of "unseen" dust mass (i.e., with particle
sizes either much larger or smaller than 1 mm), since, even
in the absence of such unseen contributions, the gas mass
inferred from mm dust measurements is in some cases al-
ready $\sim 0.2$ times the central star’s mass, and thus close
to the limit for gravitational instability. Given the evidence
that grain growth to mm and cm scales has occurred in the
outer regions of disks (Wilner et al. 2006), we are confident
that this grain size scale contains the majority of the disk
solid mass at these radii, and thus, by implication, the ma-

dority of the solid mass in the disk. Thus, while the dust seen
in the sub-mm is not contributing to the planet formation
process (because it is mainly at radii where it has not had

time to grow to large - greater than metre - size scales), we
are suggesting that it is nevertheless a good measure of the
primordial inventory of solids in the disk.

The fact that the sub-mm dust mass distribution fits
the observed planet-metallicity relation so well is because
there is an order of magnitude difference between the highest
and lowest masses of the top $\sim 25\%$ most massive gas disks
(e.g., Figs 1 and 2). This result is not specific to the Taurus-

Auriga star forming region, since class II disks in $\rho$ Oph also
exhibit an order of magnitude range for the most massive
25% of those disks (see Fig. 9 of André & Montmerle 1994).
If this distribution had been much narrower or broader then
we would have been able to rule out the critical solid mass
model.

One further requirement of nature for the critical solid
mass model to work is for a disk’s outer radius to be less im-
portant than its solid mass in setting the outcome of planet
formation. As noted in §3.1, models such as those in IL04
rely on a critical surface density (rather than mass). For the
surface density profile assumed by IL04, the surface density
normalisation ($f_{\text{in}}$, where $\Sigma \propto f_{\text{in}}$), disk outer radius ($r_{\text{out}}$) and
total solid mass ($M_s$) are related via $M_s \propto f_{\text{in}} r_{\text{out}}^{0.5}$. Thus the
mapping between critical surface density and critical mass is
(weakly) dependent on $r_{\text{out}}$. While disk radii have been measured using sub-mm interferometry (Kitamura et
al. 2002; Andrews & Williams 2007), these samples are bi-
ased toward the most massive disks so that it is not clear
how representative the observed distribution is of the pop-
ulation as a whole. However, there is no evidence that the
distribution of $r_{\text{out}}$ is as broad as that of disk masses seen
by AW05. We therefore expect the surface density of solids
in the planet formation region to be mainly controlled by
$M_s$ rather than $r_{\text{out}}$, thus explaining the apparent success of
sub-mm flux as a predictor of planet forming potential.

3.3 Disks forming detectable planets

One implication of this study is that we can predict which
of the disks in the AW05 sample will go on to form plan-
ets like those detected in the current radial velocity surveys.
The class IIs in their sample with more than 0.5$M_J$ of dust
are 04113+2758, DL Tau, GG Tau and GO Tau. However,
we disqualify GG Tau as a planet-forming candidate, since
its disk is circumbinary (Guilloteau, Dutrey & Simon 1999),
and so its high sub-mm flux does not equate with a high sur-
face density of solids in the inner disk. Massive circumbinary
disks are rare (Jensen, Mathieu & Fuller 1996), so the ma-

jority of the more massive disks are not circumbinary disks
and so would not be unsuitable for forming planets. Apply-
ing the same 0.5$M_J$ dust mass limit to the $\rho$ Oph study of
André & Montmerle (1994) indicates that of the class IIs in
this region, AS205, EL24, GSS39 and SR24S may go on to
form detectable planets.

While we do not claim that we can unambiguously pre-
dict the outcome of planet formation for any one of these
systems, we do suggest that studying the disks that are pre-
dicted to form planets, the characteristics of which we can
constrain at least statistically, may provide a valuable way
of probing the environments in which such planets form.

The fact that it is the most massive disks which go on
to form detectable planets means that these disks must be
close to being gravitationally unstable, since the ratio
$M_{\text{disk}}/M_s > 0.05$ for $Z = 0$ and $M_s = 1M_\odot$. This suggests
that instability could play a role in the formation process.
However, this cannot be the only determining factor, since

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the gravitational instability process itself is not affected by metallicity (Cai et al. 2006), and there would be no metallicity dependence if $M_{g,crit}$ is a constant and not dependent on metallicity. Thus, this suggests that some degree of instability may help speed up the core accretion process, e.g., through concentration of particles in spiral structures (Rice et al. 2004) or instability in a thin dust layer (Youdin & Shu 2002).

3.4 Observational tests

Here we suggest three observational tests of the critical solid mass model:

Firstly, if the model is correct, we would expect $P_{pl}(Z)$ to rise much less steeply with $Z$ at metallicities above 0.5 dex than implied by an extrapolation of equation (1), since at higher metallicities the model predicts that planets would be able to form in lower mass disks, and that $P_{pl}(Z)$ in this regime would reflect the disk mass distribution of intermediate mass disks. A discrepancy between the observed disk mass distribution and that resulting from an extrapolation of equation (1) to $Z > 0.5$ dex is readily apparent by considering how the solid curve on Figure 1, if extrapolated to lower disk masses, would compare with the dashed line on that Figure. Whether a suitable high metallicity sample can be found to test this prediction remains to be seen (e.g., Laughlin 2000; Valenti & Fischer 2005; Taylor 2006).

Secondly, one of the key assumptions of the model was that the distribution of protoplanetary disk masses is universal in that it is independent of metallicity. This can be tested by measuring the distribution of dust masses in low (or high) metallicity star forming regions using sub-mm photometry, since these masses should be correspondingly lower (or higher) than those of nearby regions like Taurus-Auriga (or higher) than those of nearby regions like Taurus-Auriga where $Z \approx 0$. While ALMA can detect the brightest known class II disks out to 20 kpc, we are not aware of any young (< 10 Myr) cluster within the Milky Way which has a measured metallicity that is sufficiently sub- or super-solar for the predicted difference in disk mass distribution in comparison with Taurus-Auriga to be confidently detected, although star forming clusters such as those found by Santos et al. (2000) and Yun et al. (2007) may be suitable candidates if their large Galactocentric distances (15-16.5 kpc) are indicative of a low metallicity as suggested by observations Cepheids which indicate a metallicity gradient in the Milky Way of $-0.06$ dex/kpc (Luck et al. 2006).

Thirdly, we will be able to test in due course an adjacent hypothesis, i.e., that the incidence of planets of lower masses (and at greater orbital distances) is also regulated by a (lower) critical solid mass threshold. For example, extrapolation of the exoplanet semi-major axis distribution to 20 AU suggests that surveys able to detect planets to that distance would double the fraction of stars known to have planets to 12% (Marcy et al. 2005). The simplest hypothesis we can apply to this population would simply be that the progenitor disks corresponded to the top 12% of the disk mass distribution, implying a critical solid mass of $\sim 0.3 M_J$. We plot in Figure 4 the predicted dependence of planet frequency on metallicity in this case. Although this "prediction" will eventually be compared with observational data, we emphasize that it is not entirely clear how this adjacent hypothesis (i.e., that the critical solid mass is lower for planets located at larger distances) can be squared with the expectations of core accretion models.

4 CONCLUSIONS

We have presented a simple analytical model which can be used to predict the outcome of planet formation, in which the formation of a planet that is detectable in radial velocity studies depends only on the mass of solids in the protoplanetary disk. We showed that this model predicts that the observed planet-metallicity relation is a reflection of the disk mass distribution. We also argued that the sub-mm dust mass seen in the protoplanetary disk phase is a good tracer of the initial mass budget available close to the star for planet formation, and showed that the observed planet-metallicity relation is consistent with the disk mass distribution estimated from sub-mm observations of protoplanetary disks if the critical solid mass required to form detectable planets is $0.5 M_J$.

We suggested that the detailed physics of the IL04 core accretion models boil down to a critical solid mass required to form detectable planets, although it needs to be confirmed that the good empirical agreement with the IL04 models is more than a coincidence. However, the value of this model is not just in its relevance to specific core accretion models, but in its general applicability, since it shows how the observed planet-metallicity relation would be reproduced by any planet formation model which imposes a critical solid mass for the formation of detectable planets. Other reasons for imposing a threshold on a disk’s solid mass before detectable planets can form include the possibility that such conditions are required for the formation of > km-sized planetesimals through gravitational instabilities (e.g., Johansen, Klahr & Henning 2006).

The value of this model is also in its simplicity, since
Figure 5. Distribution of infrared luminosities ($f = L_{IR}/L_*$) of the debris disks of A stars in the model of Wyatt et al. (2007). The distribution for the debris disks formed from the most massive 6% of protoplanetary disks (the planet bearers) is compared with those formed from the least massive 94% of protoplanetary disks (the non-planet bearers).

Application of this model to known systems implies that the disks of 04113 + 2758, DL Tau, GO Tau, AS205, EL24, GSS39 and SR24S will form (or have formed) gas giant planets. While the outcome of planet formation in individual systems is uncertain, we suggest that studying these disks may help constrain the physical conditions of disks in which we know, at least statistically, what the outcome of planet formation will be. Observational tests of the model include a flattening of the metallicity relation for $Z > 0.5$ dex and also a flattening as planet searches continue.

REFERENCES