Predicting the frequencies of diverse exo-planetary systems

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ABSTRACT
Extrasolar planetary systems range from hot Jupiters out to icy comet belts more distant than Pluto. We explain this diversity in a model where the mass of solids in the primordial circumstellar disk dictates the outcome. The star retains measures of the initial heavy-element (metal) abundance that can be used to map solid masses onto outcomes, and the frequencies of all classes are correctly predicted. The differing dependences on metallicity for forming massive planets and low-mass cometary bodies are also explained. By extrapolation, around two-thirds of stars have enough solids to form Earth-like planets, and a high rate is supported by the first detections of low-mass exo-planets.

Key words: circumstellar matter – planetary systems: protoplanetary discs – planetary systems: formation

1 INTRODUCTION
Extrasolar planetary systems have largely been identified by a change in the line-of-sight velocity in spectra of the host star, the ‘Doppler wobble’ method (Cumming et al. 1999, e.g.). This technique detects inner-system gas-giant planets out to a few astronomical units. Contrasting larger-scale systems are those with ‘debris’ disks, rings of dust particles produced in comet collisions, whose presence indicates that parent bodies exist at least up to a few kilometres in size (Wyatt & Dent 2002). Most images of debris disks show central cavities similar to that cleared by Jupiter and Saturn in our own Solar System (Liu & Zook 1999), and also sub-structure attributed to dust and planetesimals piled up in positions in mean motion resonance with a distant giant planet (Wyatt 2003). Beichman et al. (2003) have discovered a few systems with both debris disks and inner giants, linking these divergent outcomes.

These various planetary systems could reflect different initial states, in particular the quantity of planet-forming materials in the circumstellar disk around the young host star. In core-growth models (Pollack et al. 1996; Hubickyj et al. 2003, e.g.), a large supply of refractory elements (carbon, iron, etc.) should promote rapid growth of planetesimals that can then amalgamate into planetary cores. If gas still persists in the disk, the core can attract a thick atmosphere and form into a gas giant planet; the disk is also viscous so that the planet tends to migrate inwards (Nelson et al. 2000). For sparse refractories, however, only small comets up to Neptune-like ‘ice giants’ may have formed when the gas disperses (Greaves et al. 2004a) suggested that such systems will be characterised by planetesimal collisions and hence debris emission.

Here we classify these outcomes from most to least successful, and postulate that the dominant agent is the initial mass in refractories. We aim to test whether one underlying property has a highly significant effect on the outcome, and so intentionally ignore other properties that could affect the planetary system formed around an individual star. Such properties include the disk size, geometry, composition and lifetime, as well as the stellar accretion rate, emission spectrum and environment. Stochastic effects are also neglected, such as inwards migration caused by inter-planet encounters (Marzari & Weidenschilling 2002) and debris brightening after collisions of major planetesimals (Dominik & Decin 2003). Such factors are beyond the scope of our simple model, but are important for detailed understanding of the formation of particular kinds of planetary system.

2 HYPOTHESIS
The hypothesis explored here is that the mass of solid elements in a primordial circumstellar disk can be quantified, and linked to an outcome such as a detectable debris disk or radial velocity planet. The only piece of relevant ‘relic’ in-
formation for a particular star is the metallicity, quantified by $[\text{Fe/H}]$, the logarithmic abundance of iron with respect to hydrogen and normalised to the Solar value, and this quantity is taken here to track refractories in general. In combination with a distribution of total (gas-plus-dust) masses of primordial disks, masses of solids can then be inferred statistically. Our basic hypothesis is that the metallicity is a relic quantity originally common to the young star and its disk, and that higher values of the trace refractory iron should correlate with more effective planet growth.

It is well-known that gas giant detection rates rise at higher $[\text{Fe/H}]$ (Fischer & Valenti 2003, e.g), while Greaves et al. (2006) have noted that debris detection is essentially independent of metallicity. Robinson et al. (2006) have shown that the growth of gas giant planets can be reproduced in simulations taking disk mass and metal content into account. Here we extend such ideas to include systems with Doppler planets, with debris disks, and with both phenomena.

Metal-based trends are now identified (Figure 1) for four outcomes, ranked from most to least successful: a) hot Jupiters, b) more distant Doppler planets, with semi-major axis beyond 0.1 AU, c) systems with both giant planets and debris, and d) stars with debris only. Here we base ‘success’ (implicitly, via core-accretion models) on a large supply of refractories and so fast evolutionary timescales, with hot Jupiters rapidly building a core, adding an atmosphere and migrating substantially towards the star. In systems with progressively less success: planets form more slowly so they migrate less over the remaining disk lifetime (cool Jupiters); only some of the material is formed into planets (planet-and-debris systems); and mainly planetesimals are formed, perhaps with planets up to ice-giant size (debris). The 0.1 AU divide of hot/cool Jupiters is not necessarily physical, and is simply intended to give reasonable source counts. Notably, in systems with debris plus Jupiters, the planets orbit at $>0.5$ AU, consistent with even lower success in migration.

The plotted ranges of $[\text{Fe/H}]$ shift globally to higher values with more success, as expected – however, the ranges also overlap indicating that the fraction of metallic elements is not the only relevant quantity. The hypothesis to be tested here is that the underlying dominant factor is the total mass of metals in the primordial disk. This mass is the product of the total mass $M$ of the disk (largely in $\text{H}_2$ gas) and the mass-fraction $Z$ of refractory elements. The range of $Z$ for each outcome is broad because disks of different $M$ were initially present; thus, a massive low-metal disk and a low-mass metal-rich disk could both lead to the same outcome.

### 3 MODEL

The distribution of the product $M_S \propto M Z$ was constructed from a mass distribution measured for primordial disks (Andrews & Williams 2005) and our metal abundances measured in nearby Sun-like stars (Valenti & Fischer 2005).

With the canonical gas-to-dust mass ratio of 100 for present-day disks, the mass of solid material is then

$$M_S = 0.01 M \times 10^{[\text{Fe/H}]},$$

(1)

with the modifying $Z$-factor assumed to be traced by iron. Padgett (1996) and James et al. (2006) find that the metallicities of present-day young star clusters are close to Solar (perhaps lower by $\sim 0.1$ dex), so the reference-point values of 100 for the gas-to-dust ratio and 0 for $[\text{Fe/H}]$ are self-consistent to good accuracy.

Our model has the following assumptions.

- There are contiguous bands of the $M_S$ distribution, corresponding to different planetary system outcomes. Any one disk has an outcome solely predicted by its value of $M_S$. The ranges of $M$ and $Z$ can overlap, as only the $M_S$ bands are required to be contiguous.
- A particular outcome will arise from a set of $M Z$ products, with the observable quantity being the metallicity ranging from $Z(\text{low})$ to $Z(\text{high})$. This $Z$-range sets the $M_S$ bounds, as described below.
- For the stellar ensemble, all possible products are assumed to occur, and to produce some outcome that is actually observed. There are no unknown outcomes (novel kinds of planetary system not yet observed).
- $M$ and $Z$ are taken to be independent, as the disk mass is presumably a dynamical property of the young star-disk system and the mass in solids is always a small component.

The $M_S$ bounds were determined using the minimum number of free parameters. For a particular outcome, the observed $Z$-range is assumed to trace the bounds of $M_S$, i.e.

$$M_S(\text{high})/M_S(\text{low}) = f \times Z(\text{high})/Z(\text{low})$$

(2)

where $f$ is a constant, and this relation sets solid-mass bounds for each outcome. Results presented here are mainly for $f = 1$, but more complex relationships could exist, and other simple assumptions without further variables could also have been made – alternative forms of Eq. 2 are explored in section 4.1.

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1 The separation of hot and cool Jupiters in $[\text{Fe/H}]$ has been previously noted as having modest significance (Santos et al. 2001). Sasselov (2002); a K-S test here gives $P=0.35$ for the null hypothesis using the Fig. 1 data (errors in $[\text{Fe/H}]$ of typically 0.025 dex). The overlap of the two distributions is in fact an integral part of our model, where two factors contribute to the outcome.
Predicting the frequencies of diverse exo-planetary systems

3.1 Input data

The M-distribution is taken from a deep millimetre-wavelength survey for dust in disks in the Taurus star-forming region by Andrews & Williams (2005). Nünberger et al. (1998) have found similar results from (less deep) surveys of other regions, so the Taurus results are taken here to be generic, under the simplification that local environment is neglected. The total disk masses M were found from the dust masses multiplied by a standard gas-to-dust mass ratio of 100. The mean of the log-normal total disk masses is 1 Jupiter mass (i.e. log M = 0 in M_Jup units) with a standard deviation of 1.15 dex, and detections were actually made down to $-0.65$. The Z-distribution is from the volume-limited sample from Fischer & Valenti (2003) of main sequence Sun-like stars (F, G, K dwarfs) within 18 pc. These authors also list 850 similar stars out to larger distances that are being actively searched for Doppler planets. In the 18 pc sample, the mean in logarithmic [Fe/H] is $-0.06$ and the standard deviation is 0.25 dex.

Both M and Z distributions have an upper cutoff at 200 $M_{Jup}$ for ‘classical T Tauri’ stars, and for Z the adopted cutoff in [Fe/H] is $+0.40$ from the 18 pc sample (with a few planet-hosts of [Fe/H] up to 0.56 found in larger volumes).

3.2 Observed frequencies

The Doppler-detection frequencies quoted in Table 1 are mostly from the set of 850 uniformly-searched stars with [Fe/H] values. The statistics for hot Jupiters range from 16/1330 = 1.2 % in a single-team search (Marcy et al. 2005) up to 22/850 = 2.6 % in the uniform dataset (Fischer & Valenti 2003). For cool Jupiters (outside 0.1 AU semi-major axis), the counts are 76/850 = 8.9 % (Fischer & Valenti 2003), with Marcy et al. (2003) finding a range from 72/1330 = 5.4 % within 5 AU up to an extrapolation of 11 % within 20 AU. The upper limit is based on long-term trends in the radial velocity data, and these as-yet unconfirmed planetary systems could have [Fe/H] bounds beyond those quoted here.

The debris counts are based on our surveys with Spitzer. The debris-only statistics (Beichman et al. 2006; Bryden et al. 2006) comprise 25 Spitzer detections out of 169 target stars in unbiased surveys, i.e. 15 ± 3 % with Poissonson errors. For planet-plus-debris systems, the 3 % frequency is estimated from 6/25 detections (24 ± 10 %) among stars known to have Doppler planets (Beichman et al. 2003), multiplied by the 12 % total extrapolated planet frequency (Marcy et al. 2003). In Table 1, the planet-only rates should strictly sum to only up to 9 % if this estimate of 3 % of planet-and-debris systems is subtracted.

The null set quoted has an [Fe/H] range derived from our Spitzer targets where no debris or planet has been detected. The $M_S$(low) bound for the null set has been set to zero rather than the formal limit derived from Z(low)/Z(high), as lower values of $M_S$(low) presumably also give no presently observable outcome.

4 RESULTS

The predicted frequencies (Table 1) agree closely with the observed rates in all outcome categories. This is remarkable when very different planetary systems are observed by independent methods, and ranging in scale from under a tenth to tens of AU. The good agreement suggest that solid mass may indeed be the dominant predictor of outcome.

The model is also rather robust. In particular, because the M-distribution is an independent datum, obtaining a good match of predicted and observed frequencies is not inevitable. For example, artificially halving the standard deviation of the M distribution would yield far too many

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Table 1. Results of the planetary system frequency calculations. The ranges of solid mass $M_S$(low) to $M_S$(high) were determined based on the input data of the observed [Fe/H] range for each outcome. The total disk masses given are consistent with these two boundary conditions. The ranges of observed frequency include different surveys and/or statistical errors as discussed in the text. The predicted total is less than 100 % because of a few ‘missing outcomes’ (see text). The null set represents stars searched for both planets and debris with no detections.

<table>
<thead>
<tr>
<th>outcome</th>
<th>[Fe/H] range</th>
<th>solid mass</th>
<th>total disk mass</th>
<th>predicted frequency</th>
<th>observed frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>hot Jupiter</td>
<td>$-0.08 +0.39$</td>
<td>1.7-5 [500–1600]</td>
<td>70–200 &amp; 1 % &amp; 2 ± 1 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cool Jupiter</td>
<td>$-0.44 +0.40$</td>
<td>0.24–1.7 [75–500]</td>
<td>10–200 &amp; 8 % &amp; 9 (5-11) %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>planet &amp; debris</td>
<td>$-0.13 +0.27$</td>
<td>0.10–0.24 [30–75]</td>
<td>5–30 &amp; 5 % &amp; 3 ± 1 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>debris</td>
<td>$-0.52 +0.18$</td>
<td>0.02–0.10 [5–30]</td>
<td>1–30 &amp; 16 % &amp; 15 ± 3 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>null</td>
<td>$-0.44 +0.34$</td>
<td>&lt; 0.02 [&lt; 5]</td>
<td>&lt; 15 &amp; 62 % &amp; ~ 75 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Absolute values for $M_S$ were derived iteratively, working downwards from the most successful outcome. The upper bound for the hot Jupiter band was derived from the highest disk mass observed and highest metallicity seen for this outcome (under the assumption that all M Z products are observed). Other outcomes were then derived in turn assuming $f = 1$, and working down in order of success, with each lower bound $M_S$(low) setting the upper bound $M_S$(high) for the next most successful outcome. Both M and Z have log-normal distributions, and 100 equally-likely values of log(M) combined with 1000 equally-probable values of log(Z), i.e. [Fe/H].

2 Figure 1 also includes a few prior-candidate systems (www.roe.ac.uk/ukatc/research/topics/dust/identification.html) confirmed by Spitzer programs.
stars with detectable systems (75%), and many more cool Jupiters than hot Jupiters (around 20:1 instead of 5:1). The model is also reasonably independent of outlier data points. For example, adding in the low-metallicity system neglected in Figure 1 would raise the prediction for cool Jupiters from 8% to 12%, or extending the upper Z-cutoff to the [Fe/H] of the distant Doppler systems would raise this prediction to 11%. However, the resulting effects on the less-successful system probabilities are less than 1%. This suggests that although small number statistics may affect the predictions, the results would not greatly differ if large populations were available, that could be described by statistical bounds rather than minimum- to maximum-[Fe/H].

The model also accounts for nearly all outcomes, as required if each $M_S$ is to result in only one planetary system architecture. For a few $M_S$ products, one of the outcomes would be expected except that the Z value involved lies outside the observed ranges. These anomalous systems sum to 9% (hence the Table 1 predictions add to <100%). These ‘missing’ systems are predominantly debris and debris-plus-planet outcomes. The latter class has the smallest number of detections (Figure 1), and more examples are likely to be discovered with publication of more distant Doppler planets.

A further check is that the disk masses are realistic, in terms of producing the observed bodies. Doppler systems are predicted here to form from 5-200 Jupiter masses of gas in the disk, enough to make gas giant planets. Also, the $M_S$ values of 30-1600 Earth-masses could readily supply the cores of Jupiter and Saturn (quoted by Saumon & Guillot (2004) as $\approx 0 - 10$ and $\approx 10 - 20 M_\oplus$ respectively) or the more extreme $\sim 70 M_\oplus$ core deduced for the transiting hot Jupiter around HD 149026 (Sato et al., 2005). Similarly, populations of colliding bodies generating debris have been estimated at $\sim 1$ - 30 Earth-masses (Wyatt & Dent, 2002; Greaves et al., 2004), a quantity that could reasonably be produced from 5-75 Earth-masses in primordial solids (Table 1).

To make Jupiter analogues within realistic timescales, core growth models (Hubickyj et al., 2003, e.g.) need a few times the Minimum Mass Solar Nebula, which comprised approximately 20 $M_{Jup}$ (Davis, 2003). The Solar System itself would thus have contained a few times 0.2 $M_{Jup}$ in solids, of which 0.15 $M_{Jup}$ (50 $M_\oplus$) has been incorporated in planetary cores. These primordial mass scales would place the Solar System in the cool Jupiter category, and this is in fact how it would appear externally (the dust belt being more tenuous than in detected exo-systems).

4.1 Alternative models

Equation (2) was further investigated with $f \neq 1$. Reasonable agreement with the observations was obtained only for $f$ in a narrow range, $\approx 0.8 - 1.1$. For higher $f$, systems with debris are over-produced, while for lower $f$ there are too few Doppler detections. This suggests that the hypothesis that the Z-range directly traces the $M_S$ range for the outcome to occur is close to correct, although not well understood. Qualitatively, there is a locus of points in $M$, Z parameter space that lie inside the appropriate $M_S$ bounds for an outcome, and at some mid-range value of M it is likely that all the Z(low) to Z(high) values are appropriate, and so this traces the product $M_S$(high) to $M_S$(low) (provided no more extreme values of Z are suitable at the extrema of M). Z seems to have the most constrictive effect on outcome because the distribution is much narrower than that of M, and thus if M changes by a large amount, there is no corresponding value of Z than can preserve a similar $M_S$. Simulations of planetesimal growth as a function of mass of solids in the disk are needed to further explore why the ranges of Z and $M_S$ match so closely.

One even simpler model was tested, with a constant range of $M_S$(high)/$M_S$(low) for each observable outcome. Assuming that $\sim 1 M_\oplus$ of solids is needed for the minimum detectable system (planetesimals creating debris), and that the most massive disks contain 1600 $M_\oplus$ of solids (from the maximum M and Z), then this mass range can be divided into equal parts for the four observable outcomes, with a range of log-M $= 0.8$ for each. This simple model fails, in particular greatly over-producing debris-and-planet systems at 12%. Thus, it seems that the Z-range does in fact contain information on the outcomes.

4.2 Distributions within outcomes

The dependences on metallicity of planet and debris detections arise naturally in the model. Doppler detections are strongly affected by metallicity: [Fischer & Valenti (2003)] find that the probability is proportional to the square of the number of iron atoms. Our model predicts P(hot Jupiter) $\propto$ N(Fe)$^{2.7}$ and P(cool Jupiter) $\propto$ N(Fe)$^{0.9}$, above 80% and 30%-Solar iron content respectively. The observed exponent of 2 for all planets is intermediate between the two model values, and as predicted by [Robinson et al. (2000)], a steeper trend for short-period planets is perhaps seen, at least at high metallicities (Figure 1). These dependences arise because large solid masses are needed for fast gas giant formation with time for subsequent migration, and so when [Fe/H] is low a large total disk mass is required, with rapidly decreasing probability in the upper tail of the M-distribution.

In contrast, the model predicts a weak relation of P(debris) $\propto$ N(Fe)$^{0.2}$ (for 30-160% Solar iron), agreeing with the lack of correlation seen by [Bryden et al. (2000)]. As fewer solids are needed to make comets, a wide variety of parent disks will contain enough material, so little metal-dependence is expected. For example (Table 1), $M_S$(low) can result from a 6 $M_{Jup}$ disk with [Fe/H] of $-0.5$ or a 1.2 $M_{Jup}$ disk with [Fe/H] of $+0.2$. As these masses are both central in the broad M-distribution, they have similar probability, and so the two [Fe/H] values occur with similar likelihood.

5 DISCUSSION

The excellent reproduction of observed frequencies and dependences on metallicity both suggest that the method is robust. Hence, rather surprisingly, the original search for a single parameter that has a dominant effect on outcome has succeeded. Under the assumption that the metallicity ranges track the different outcomes, the primordial solid mass of the circumstellar disk is identified as this parameter.

One implication is that for an ensemble of stars of known metallicity, the proportions of different kinds of planetary system may be predicted, without the need for detailed models of individual disks. As main-sequence stars retain no relic information on their primordial disk masses, this
Distribution of solid masses. The shaded areas represent (from right to left) systems producing hot Jupiters, cool Jupiters, gas giants plus debris disks, debris only, and disks with 1 Earth-mass or more of solids. Disks further to the left have no presently predicted observable outcome.

is a very useful result, leading to estimates of the frequency and variety of planetary systems, for example among nearby Sun-analogues of interest to planet-detection missions.

The model may also be used to examine other planetary system regimes that have just opened up to experiment. For example, transiting hot Jupiters have not been detected in globular clusters, although the stellar density allows searches of many stars. In 47 Tuc, no transits were found amongst ~20,000 stars, although 7 detections would be expected based on the typical hot Jupiter occurrence rate \cite{Weldrake+2002}. Assuming these old stars formed with disks following the standard M-distribution, but with metallicities of only one-fifth Solar in a narrow range with $\sigma \approx 0.05$ dex \cite{Caretta+2004}, then the model predicts that less than 1 in $10^6$ disks can form a hot Jupiter – the solid masses are too small for fast planet growth and subsequent migration.

Finally, an upper limit can be estimated for the number of young disks that could form an Earth-mass planet. The maximum fraction (Figure 2) is set by disks of $M_s \geq 1 M_{\odot}$, summing to two-thirds of stars. Thus one in three stars would not be expected to host any Earth-analogue, and irrespective of metallicity since the occurrence of this minimum mass is rather flat with $P \propto N(\text{Fe})^{0.25}$. However, if the upper two-thirds of disks may be able to form terrestrial planets, this would agree with the large numbers predicted by the simulations of \cite{Ida&Liz2004}, and also with the first planetary detections by the microlensing method. Two bodies of only around 5 and 13 Earth-masses have been detected around low-mass stars, and \cite{Gould2006} estimate a frequency of $0.37 \pm 0.21, +0.30$ in this regime of icy planets orbiting at $\sim 3$ AU. Our model finds that $40\%$ of stars have disks with $M_s$ greater than 5 $M_{\odot}$, so the minimum materials to form such planets are present at about the observed frequency. While very preliminary, this result supports the prediction that many stars could have low-mass planets.

6 CONCLUSIONS

We tested the hypothesis that a single underlying parameter could have the dominant effect on the outcome of planetary system formation from primordial circumstellar disks. An empirical model with the mass of solids as this parameter produces a very good match to the observed frequencies and to the dependence on host-star metallicity, when the metallicity range within each outcome is used to estimate the solid-mass range. This model may be very useful for making statistical predictions of planetary system architectures for ensembles of stars of known metallicity, such as nearby Solar analogues of interest to exo-Earth detection missions.

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