Origin of the Moon

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The Earth-Moon system

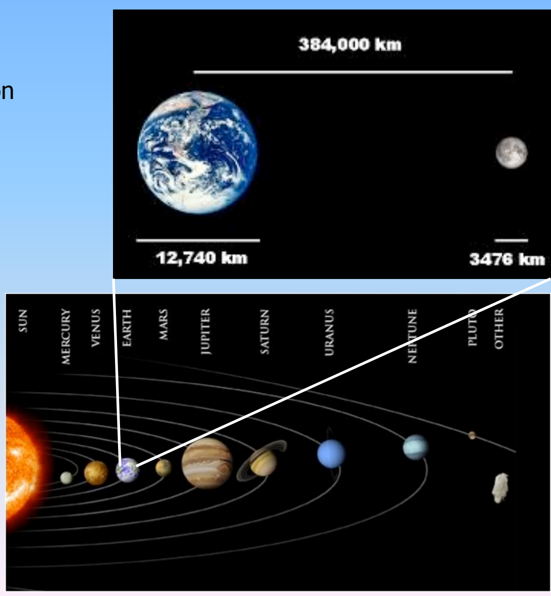
The Moon orbits the Earth at
a_{moon} = 385,000 km
with an eccentricity of 0.05, inclination to ecliptic of 5°

The Earth orbits the Sun at
a_{earth} = 150,000,000 km

Earth's Hill sphere (the distance at which objects are no longer gravitationally bound) is at

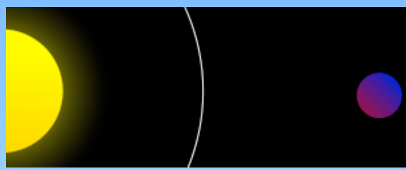
$$R_{hill} = a_{earth} (M_{earth} / 3M_{sun})^{1/3} = 1{,}500{,}000 \text{ km}$$

So Moon is well within this limit at R_{hill} / 4, though note that orbits beyond R_{hill} / 2 are unstable



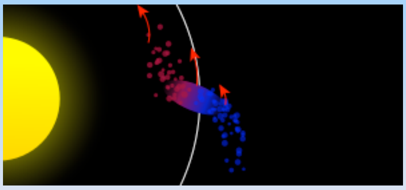
Roche radius

The Roche radius is the distance at which tidal forces on a satellite are greater than its self-gravity and so would tear it apart



For a solid satellite

$R_{roche} = 1.26\, R_{earth}\, (\rho_{earth} / \rho_{satellite})^{1/3}$
$= 9{,}500$ km

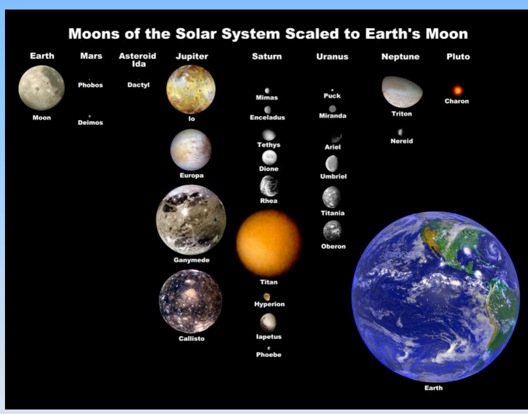


For a fluid satellite

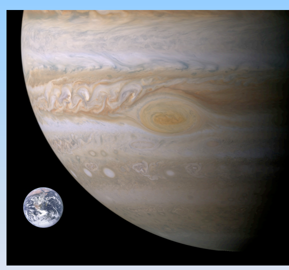
$R_{roche} = 2.44\, R_{earth}\, (\rho_{earth} / \rho_{satellite})^{1/3}$
$= 18{,}400$ km

The Moon is ~20x beyond these limits

The Moon compared with other moons



There are other moons that are bigger than our Moon, but these orbit giant planets that are much bigger than the Earth



Our Moon is large compared with the size of the parent planet: $M_{moon} = M_{earth} / 80$
Other moons all have mass ratios $< M_{pl} / 4000$
… apart from Charon which is half the size of Pluto (and $M_{charon} = M_{pluto} / 8$)

Angular momentum in Earth-Moon

Orbital angular momentum:

$J_{orbEM} \sim M_{moon} [G\, M_{earth}\, a_{moon}]^{1/2}$ (plus M_{moon}/M_{earth} and e_{moon} terms)
$= 2.9 \times 10^{34}$ kg m^2 / s

Rotational angular momentum of solid homogeneous body:

$J_{rotE} \sim 4\pi\, M_{earth}\, R_{earth}^2 / 5\, P_{rot}$ (but most objects have higher density cores)
$= 7.1 \times 10^{33}$ kg m^2 / s

$J_{rotM} \sim J_{rotE} / (80 * 3.7^2 * 27) = J_{rotE} / 30{,}000$ so is negligible

So, most of the angular momentum of the Earth-Moon system is in the orbital motion, which is in contrast to other moons in the solar system (e.g., for Jupiter's moons $J_{orb} < J_{rot} / 100$)

Tides

Gravity of Moon affects shape of Earth (e.g., moving oceans) causing dissipation of energy. Consider total energy and angular momentum, ignoring rotation of Moon:

Energy...

$$E = \underbrace{-\frac{GMm}{2a}}_{\text{orbit}} + \underbrace{\frac{1}{2}I\Omega^2}_{\text{rotation of Earth}}$$

...is dissipated by tides

$$\dot{E} = \frac{GMm}{2a^2}\dot{a} + I\Omega\dot{\Omega} < 0$$

Angular momentum...

$$J = Mm\sqrt{\frac{Ga}{M+m}} + I\Omega$$

... is conserved

$$\dot{J} = 0 = \frac{Mm}{2}\sqrt{\frac{Ga}{M+m}}\frac{\dot{a}}{a} + I\dot{\Omega}$$

Combining gives

$$\dot{E} = I\dot{\Omega}(\Omega - \omega) < 0$$

Tides cause

$$\dot{E} = I\dot{\Omega}(\Omega - \omega) < 0$$

Earth's spin is slowing (days are lengthening by 23μs/year)

Moon's orbit is receding at a rate 38mm/year

Eventually would cause Earth's spin rate to equal the orbital angular freq $\Omega=\omega$ (as then $dE/dt=0$)

This already happened to the Moon's spin (its rotation period equals its orbital period of 27 days), and means Moon keeps same face to us

Where did the Moon start?

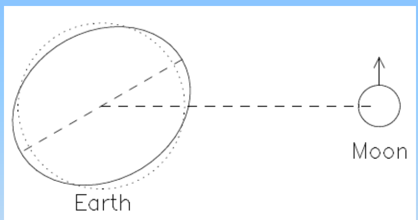
In past Earth was spinning faster and Moon was closer to the Earth; constant recession over 4.5Gyr would imply Moon started at 214,000 km

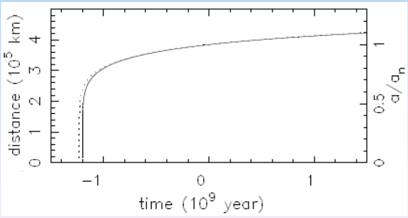
Actually dissipation would have been faster when closer, with simple model of a bulge leading the motion of the Moon giving

$da/dt \sim a^{-7} (P_{orb}/P_{rotE} - 1)$

leading to tidal catastrophe

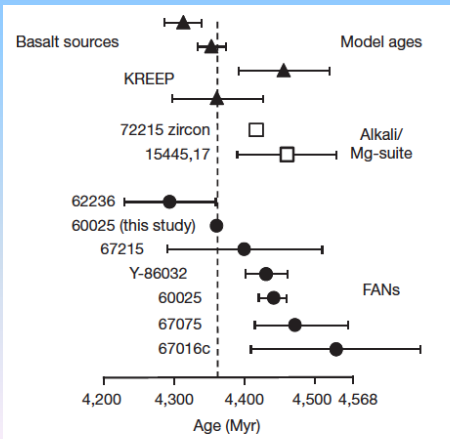
Also tidal energy loss comes out as heat (which is why Io is so volcanic), so tidal dissipation would have melted Earth



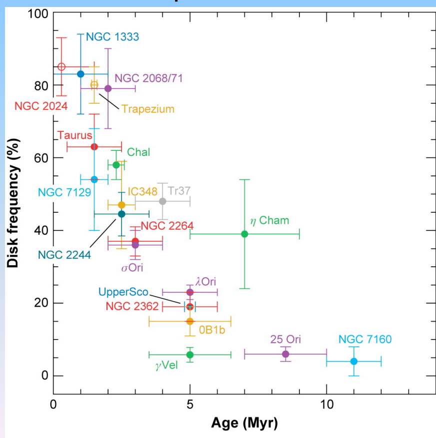


When did the Moon form?

Studies of lunar rocks give oldest ages at 30-100Myr after the Solar System formed



Protoplanetary disks disperse over ~5Myr, so Moon formed after disk dispersal, and also after meteorites and terrestrial planets formed

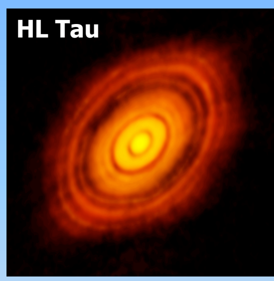


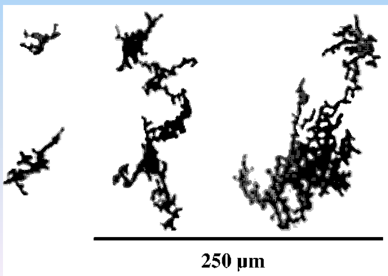
Terrestrial planet formation: stage 1

Stars are born with protoplanetary disks made of gas and μm-sized dust

Experiments show that dust grains stick to each other when they collide at anticipated velocities, and that growth to cm-size is easy

But growth beyond metre-sizes is prevented by bouncing and strong radial drift



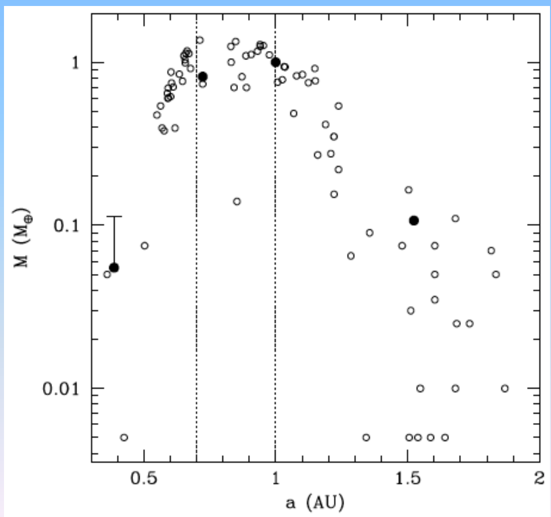


Terrestrial planet formation: stage 2

As soon as km-sized planetesimals form, it is easy to grow them into planets

They undergo runaway growth due to gravitational focussing, then oligarchic growth

Formation of something that looks like the Solar System's terrestrial planets is relatively easy, albeit with some restrictions (e.g., mass of Mars, low eccentricities)



Constraints on Moon formation

Mass: $M_{moon} = M_{earth} / 80$

Angular momentum: High J_{EM} / M_{EM} compared with other planets

Age: ~50Myr

Lack of volatiles: very dry (no water except from comets?)

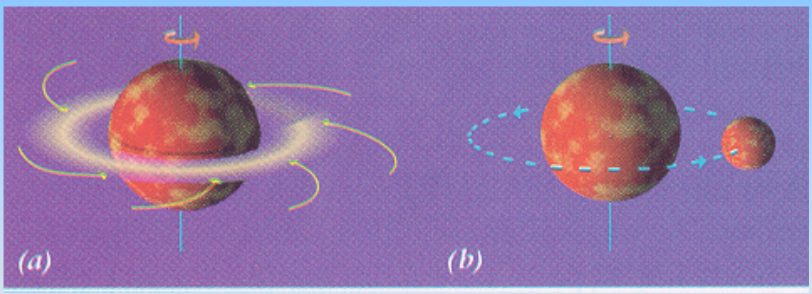
Lack of Iron: density is 3.3g/cm^3 implies 0.25x cosmic abundance of Fe, much less than Earth

Oxygen isotopes: ^{17}O / ^{18}O are identical to Earth, but these vary with position in the Solar System and so in protoplanetary disk

Magma ocean: Apollo rocks showed that Moon melted early in history forming a low density crust, denser mantle, maybe metallic core

Formation scenarios: Co-accretion

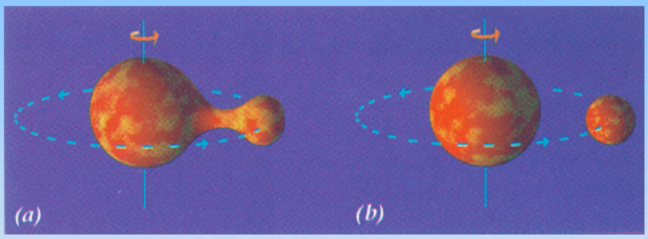
Idea: During accretion of the Earth, a circumterrestrial disk of planetesimals was formed out of which the Moon accreted



Problems: How could Earth acquire a disk with such high angular momentum? Age of Moon. Chemical composition would be same as Earth

Formation scenarios: Fission

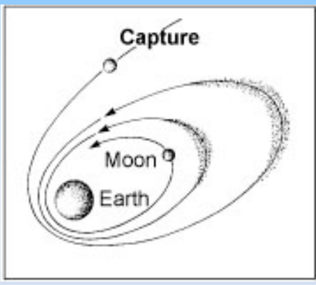
Idea: Rapidly rotating Earth undergoes fission, perhaps triggered by Solar tides, whereupon Moon receded from Earth due to tides



Problems: Dynamically implausible, viscosity damps resonant motion supposed to trigger fission

Formation scenarios: Capture

Idea: The Moon was a planetary embryo formed in a different (but nearby) part of the Solar System which was captured into orbit around the Earth

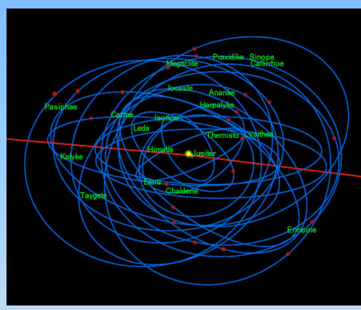


Problem: Low Fe of Moon, more likely to be captured on wide orbit (and requires third body to take energy away), no heating of Moon

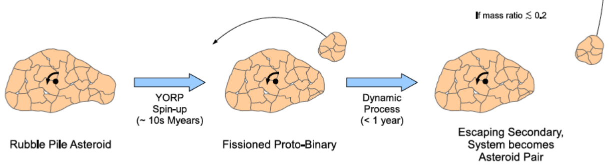
All scenarios have precedents

Jupiter's regular moons thought to have formed in a proto-jovian disk

Its irregular satellites are thought to be captured asteroids and comets



Binary asteroids may have formed by fission



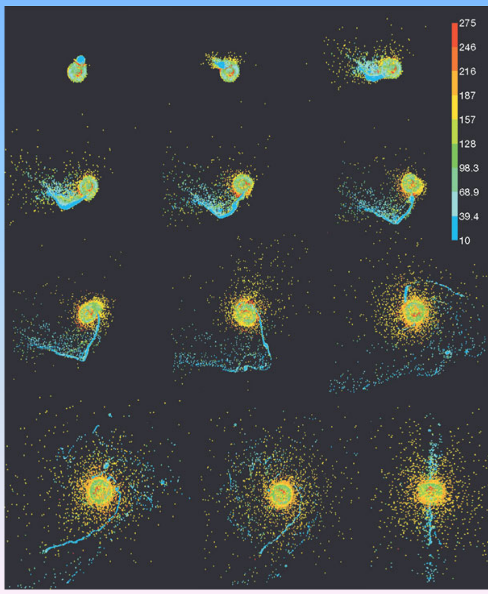
But these are different to the Moon

Formation scenarios: Giant Impact

Idea: Solve the problems of the co-accretion scenario by creating a circumterrestrial disk in a collision with a Mars-sized impactor (Theia) when Earth was 90% of its current mass

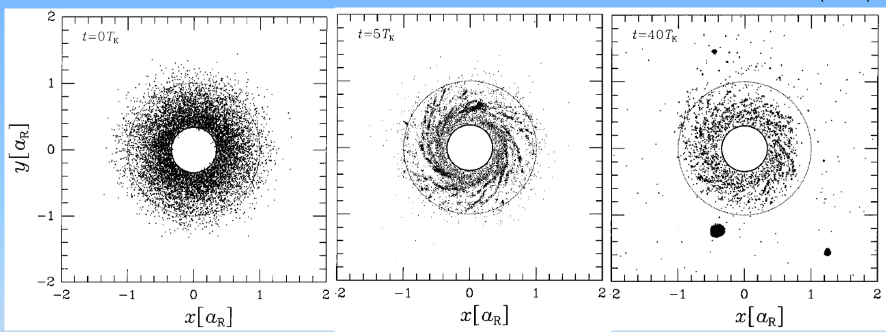
If Earth was differentiated then explains lack of Fe in Moon since this formed from mantle

Smoothed Particle Hydrodynamics (SPH) simulations show that the formation of such a disk is plausible (Canup et al. 2001)



Evolution of circumterrestrial disk

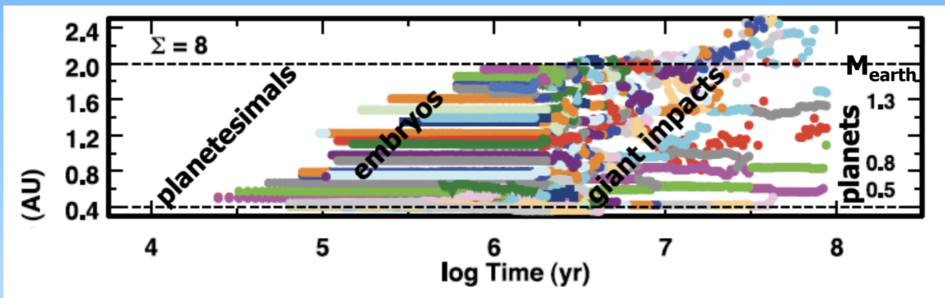
Kokubo et al. (2000)



- Disk contracts through collisional damping
- Particle clumps grow inside R_{roche} but shear out to form spiral structure
- Gravitational torques push particles beyond R_{roche} where moonlets form
- Moonlets coalesce; lunar seed sweeps up all particles pushed $>R_{roche}$
- When Moon large enough, pushes inner disk onto Earth; takes ~1month

If a circumterrestrial disk forms with 2-4 times lunar mass within the Roche radius, then an object like the Moon will coalesce out of it

Plausibility of collision

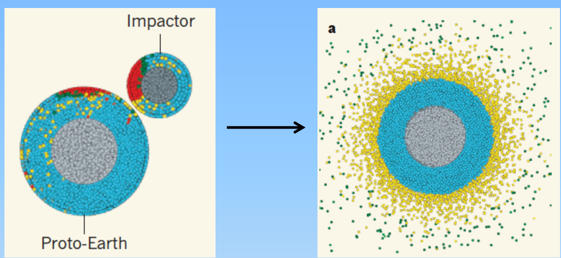


Collisions expected during final stages of formation of terrestrial planets, but requires high impact parameter and mass ratio to strip mantle, ~1% chance?

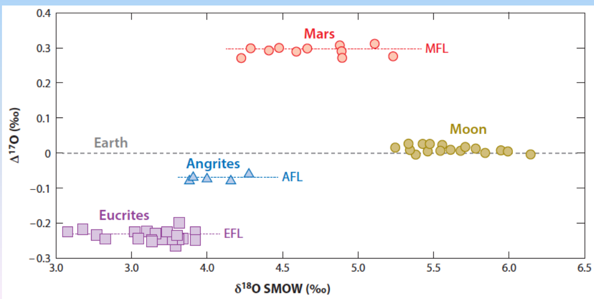
Appeal to **anthropic principle**? – if the Moon's existence favours the development of life then more likely to be observing from a planet with a Moon

Niggling composition concerns

Part of the impactor always goes into the circumterrestrial disk



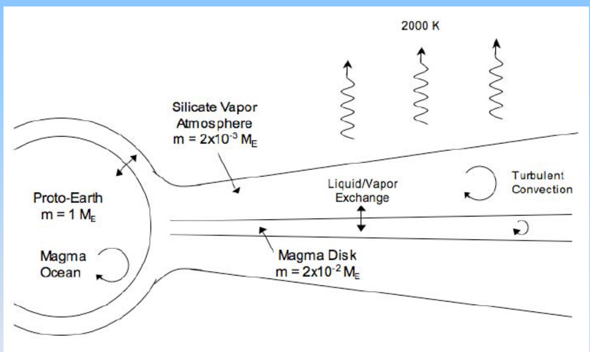
So why is isotopic composition of Moon so similar to that of the Earth?



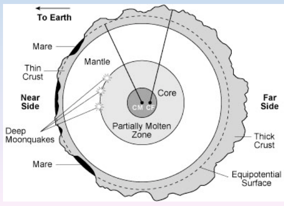
Is it likely that even a nearby embryo would have such similar composition? Perhaps 20% probability
(Mastrobuono-Battisti et al. 2015)

Explanation 1: Protolunar disk physics

Diffusion of Earth and protolunar isotopic systems through disk atmosphere (Pahlevan & Stevenson 2007)



But, required diffusion may be self-limiting due to mass and angular momentum transfer, K enrichment in lunar rocks not explained by this model but requires collision that vaporises all mantle and magma disk (Wang & Jacobsen 2016)

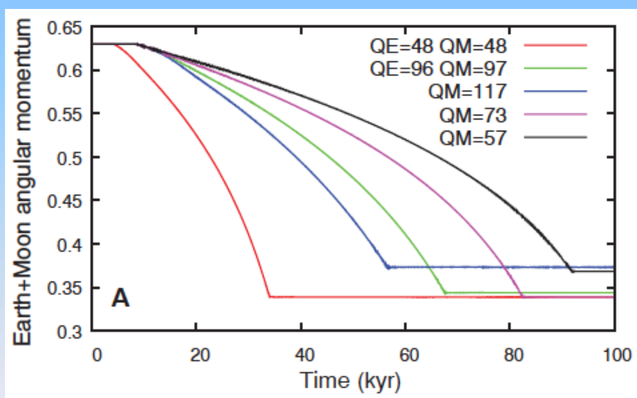


Similar idea has the equilibriation only occurring for the last moonlets accreted onto the Moon giving a late veneer of isotopically similar material (Salmon & Canup 2012)

Explanation 2: Different collision parameters

Not all collisions end up with Theia in protolunar disk, but most also end up with a system with too much angular momentum

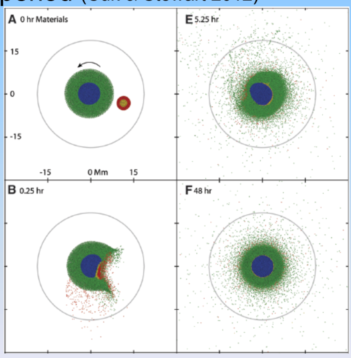
Invoke capture of Moon in *evection* resonance (Cuk & Stewart 2012) which can halve angular momentum in Earth-Moon system by exchanging it within the Sun-Earth-Moon system



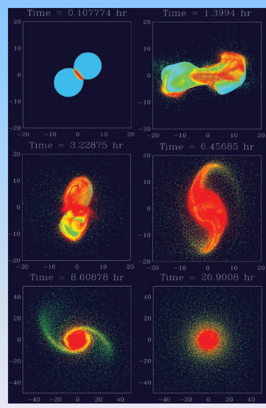
Evection resonance is between the Moon's orbital precession period and the Earth's orbital period

Explanation 2: Different collision parameters

EG1: 20km/s collision of $0.05M_{earth}$ impactor onto proto-Earth spinning close to rotational instability at 2.5hr period (Cuk & Stewart 2012)



EG2: 4km/s collision of two $0.5M_{earth}$ planets (Canup 2012)



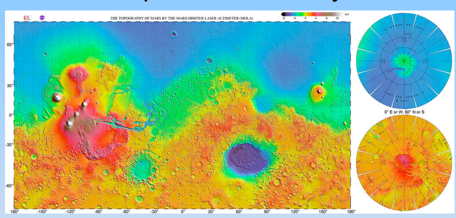
But could Earth be spinning that fast (probably requires previous giant impact), and would embryos be similar enough in mass?

Or did the Moon grow from multiple impacts (Rufu et al. 2017)?

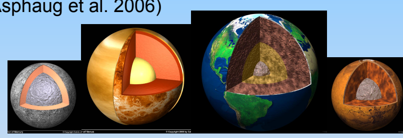
Other giant impacts in the Solar System

While origin of Moon is unsolved, all theories invoke a giant impact(s) at ~50Myr, and giant impacts appear to be a common feature in the Solar System

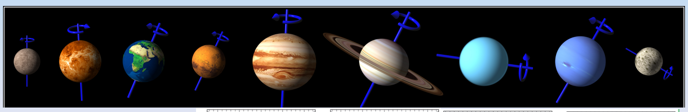
Mars hemispheric dichotomy (Marinova et al. 2008)



Mercury Fe-rich composition
(Asphaug et al. 2006)



Uranus tilt
(Kegerreis et al. 2018)



Pluto – Charon system (Canup 2011)