## **Introduction to Astrophysics**

### Michaelmas Term, 2020: Prof Craig Mackay

#### Module 4:

• White dwarfs. The discovery of Sirius B, pressure from degenerate electrons, the mass – radius relation, the Chandrasekhar mass limit, cooling times, ages, radii from gravitational redshifts.

• Neutron stars. Pressure support from degenerate neutrons, sizes of neutron stars, other extreme properties, discovery of pulsars, the standard model of a pulsar.

• Pulsars observations. Pulse properties, proper motions, masses, internal structure, pulse timing, magnetic fields, using pulsars to probe the ISM and globular clusters and test GR, recent discoveries.

## **Discovery of Sirius B**

- Sirius is the brightest star in the sky with V = -1.5. It is a main sequence star with T<sub>e</sub> ~ 10,000K and is 2.6pc away (measured via parallax).
- In 1844 it was suspected that Sirius had a companion because it appeared to wobble.
- In 1862 Sirius B was directly observed as a V=8.4 companion a few arcsec away.
- In 1915 spectroscopy implied that  $T_e \sim 27,000$ K. Stefan-Boltzman law  $\rightarrow R \sim 0.008 R_{sun}$
- But from the observed orbit, the mass was found to be 0.98M<sub>sun</sub> implying <ρ>=2.7×10<sup>9</sup> kgm<sup>-3</sup>. At that time the existence of such a hot, massive, dense star was a complete mystery.

### What is a White Dwarf star?

- Mass: 0.6 < M < 1.4 solar masses
- Size: similar to the Earth (hence term "dwarf") therefore extremely dense.
- Endpoint of stellar evolution for main sequence stars with M < 8 solar masses.
- Pressure support: degenerate electrons.
- No energy generation simply a cooling remnant. Term "white" comes from the hot (T~25,000K) temperature of the first ones discovered.
- Composition: Helium or Carbon (not Hydrogen)

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### White Dwarfs: Degeneracy Pressure

- The Pauli exclusion principle requires that no two electrons have the same quantum state, so as the density of matter increases the electrons are forced into higher and higher energy levels. The electrons have to occupy different "cells" in phase space (a space comprising the 3 spatial dimensions and 3 velocity dimensions). As they are squeezed together in the ordinary 3 dimensional space they have to move to extreme velocity space positions to not violate Pauli's exclusion principle. One consequence of this behavior is that the <u>average</u> energy per electron in a many-electron atom is considerably higher than the energy of the lowest orbit, or the average energy ~KT of a free electron.
- The same reasoning applies to a solid, like a metal, in which the electrons are essentially shared in an electron "sea." Here the energy levels merge into bands, but once again the average electron energy is higher than the energy of a free electron, and the corresponding pressure is also higher.
- A degenerate gas is one in which all of the electrons are in the lowest-energy states allowed by the Pauli Exclusion Principle, and the pressure exerted by these electrons is called degeneracy pressure.
- Heisenberg's Uncertainty Principle states that  $\Delta p \Delta x \ge \eta$  or  $\Delta p \ge \frac{\eta}{\Delta x}$
- Consider what this means...as the electron density rises, the average distance between electrons Δx shrinks and the momentum uncertainty grows. This means that some electrons will be moving very rapidly, and the pressure they exert will be correspondingly large.
- We estimate the pressure as follows. Consider a box of electrons with number density n.

#### White Dwarfs: Degeneracy Pressure

 $p_r$ 

• The pressure is equal to the momentum transferred to a unit area of the walls of the container every second by the particles in the box:

$$P = nv_x p_x$$

The average spacing between electrons must . be about: 1

$$\Delta x \approx \frac{1}{n^{1/3}} = n^{-1/3}$$



The uncertainty principle tells us:

$$\approx \frac{\eta}{\Delta x} \qquad \text{so} \quad p_x = \eta n^{1/3}$$

But since the velocity of an electron is related to its momentum by

$$v_x = \frac{p_x}{m_e} = \frac{\eta n}{m_e}$$

η

Then (in the non-relativistic case), we have:  $P = nv_x p_x = \frac{\eta^2 n^{5/3}}{m_e}$ 

### White Dwarfs: Degeneracy Pressure

- We have just derived a formula for electron degeneracy pressure. A more precise derivation (see the stars course) would have added a factor of two or so to the answer.
- From this equation of state we see that the degenerate pressure is proportional to the five thirds power of the density in the nonrelativistic case.
- Note that degeneracy pressure is independent of temperature, unlike gas pressure.
- As the electron velocity becomes relativistic and v ~ c we find that the degeneracy pressure in the relativistic case becomes proportional to density to the four thirds power.
- At terrestrial temperatures and densities and even at temperatures and densities characteristic of the solar interior – gas pressure far exceeds degeneracy pressure. However, as the number density rises, the degeneracy pressure will eventually exceed the gas pressure, and this is what happens inside a white dwarf.

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### White Dwarfs: Mass-Radius Relation

• Let us examine the mass-radius relationship for white dwarfs by working out the total energy of the system.  $E_{tot} = NE_{deg} + E_{grav} \qquad \text{Use}E_{deg} = \frac{1}{2}m_e v_x^{2} \text{ and } v_x = \hbar \frac{n^{\frac{1}{3}}}{m_e}$ 

Vol=V=R<sup>3</sup> N is total number of electrons

$$E_{tot} = NL_{deg} + L_{grav} = 0.5L_{deg} - \frac{2}{2}M_e v_x = 0.04$$
for  $v_e << c, E_{tot} = \left(\frac{\hbar^2}{2m_e}\right) \left(\frac{N}{V}\right)^{\frac{2}{3}} N - \frac{GM^2}{R}$ 
Put  $N = \frac{M}{M_p} (M = Mass of White Dwarf)$ 

$$E_{tot} = \left(\frac{\hbar^2}{2m_e}\right) \left(\frac{M}{M_p}\right)^{\frac{5}{3}} \frac{1}{R^2} - \frac{GM^2}{R}$$

• In order to find a minimum total energy we differentiate. This gives us:

$$M \approx \frac{\hbar^6}{M_e^3 G^3 m_p^5} R^{-3} \qquad \qquad N = M/m_p \text{ for } H$$
$$N = M/2m_p \text{ for } He$$

- This tells us that as the mass M increases, the radius R decreases, and the density increases.
- On the other hand, in the relativistic case.

Then 
$$E_{tot} = \hbar c \left(\frac{M}{M_p}\right)^3 \frac{1}{R} - \frac{GM^2}{R}$$

Use E<sub>DEG</sub>=pc for the energy of an electron

- Both terms on the right-hand side of this equation have the same dependency on radius R and so there is no minimum in the total energy.
- This implies that the system is unstable to collapse (or indeed to expansion)

### White Dwarfs: Mass-Radius Relation

- The system will collapse as soon as the total energy is negative.
- This tells us that the maximum mass for a white dwarf occurs when this total energy is equal to zero. This maximum mass is called the Chandrasekhar limit or Chandrasekhar mass and is given by:

When made of hydrogen	$M = \left(\frac{\hbar c}{G}\right)^{\frac{3}{2}} \frac{1}{m_p^2}$	=1.85M <sub>sun</sub> (should be 5.6M <sub>sun</sub> for H)

• The Chandrasekhar mass for white dwarfs made of He or heavier atoms is then:  $M_{1} \approx 1.4M_{2}$ 

 $M_{ch} \approx 1.4 M_{sun}$ 

When calculated properly.



Figure 10.1 Relation between the mass and radius of white dwarfs.

#### White Dwarfs: Basic Properties

- Normal stars are cooled because photons are emitted from the surface of the star.
- The conduction of heat from the centre of the star is a relatively slow process of transport from the interior to the outside by radiation and convection.
- In white dwarfs, however, degenerate electrons can travel very long distances before losing energy in a collision with the nucleus since the vast majority of the low-energy electron states are already occupied.
- This means that the energy transport mechanism in a white dwarf is electron conduction rather than radiation and it is so efficient that the interior of a white dwarf is essentially isothermal.
- At the surface of the star there is a very steep temperature gradient because the outer layer is always going to be a layer of non-degenerate gas (principally of hydrogen or helium), although this shell may be very thin indeed.
- The classification and spectra of white dwarfs reflects these conditions at different stages of evolution.
- Type DA has hydrogen lines, type DB has helium lines and type DC have an essentially featureless spectrum.
- One last type, type DZ shows lines of other elements such as calcium and this is because the material in the outer shell of these stars is accreted from the interstellar medium or from another companion star.

## White Dwarfs: Cooling Physics

- At this stage in the life of a white dwarf there are no nuclear energy sources. The power comes from the thermal energy of the particles moving within the degenerate core of the star.
- The thermal energy of each nucleus is given by:
- And total energy (  $T \sim 2.10^7$  K) is then approx:

$$\approx \frac{3}{2}kT$$
$$\approx \frac{M}{2}.\frac{3}{2}$$

- The number of nuclei in the star is equal to the mass of the star divided by the mean atomic mass of the nuclei, A with it multiplied by the mass of a proton, m<sub>p</sub>.
- The cooling time, equal to the thermal energy divided by the luminosity of the star gives us  $\tau_{cool} \sim 10^8$  years.
- In practice it turns out that the stellar atmosphere generally acts as an insulating blanket significantly increasing the cooling time because of its opacity.
- You will see that (later, this lecture):

$$T(t) = T_o (1 + \frac{5}{2} \frac{t}{\tau_o})^{\frac{-2}{5}}$$
$$L(t) = L_o (1 + \frac{5}{2} \frac{t}{\tau_o})^{\frac{-7}{5}}$$

- As the star cools its luminosity decreases and the cooling timescales become much longer. In practice the cooling takes > 10<sup>10</sup> years.
- Eventually the white dwarf turns into a giant crystal lattice in the sky radiating very slowly as its temperature gets lower and lower.
- Observations of white dwarfs let us estimate the age of the galactic disk, globular clusters and the universe in general but this requires a census of the stars to very faint mags, ~  $10^{-5}$  L  $_{\odot}$ .



### **Estimating Ages from White Dwarf Statistics**

• We search a globular cluster for very faint blue stars. These pictures show data taken with the Hubble space telescope to identify a white dwarf population in the globular cluster M4.



#### The Hertzsprung-Russell diagram for the globular cluster M4 shows the separate white dwarf cooling sequence.

#### The diagrams on the right have the theoretical cooling curves for stars of different masses overlaid on the diagram.

### White dwarf cooling

It can be shown that the luminosity  $L_{WD}$  of a white dwarf is related to its internal temperature T<sub>c</sub> by

where

$$L_{wd} = CTc^{7/2}$$

$$C = \frac{4D^3 16\Pi ac GM_H}{51K_o k} . \mu M_{wD}$$

$$= 7.3 \times 10^5 \left(\frac{M_{wD}}{M_{sum}}\right) \frac{\mu}{Z(1+x)}$$

T

This result assumes the white dwarf is insulated by a hydrogen atmosphere.

Using  $L_{WD} = 0.03 L_{sun}$ ; X = 0 (H), Y = 0.9 (He) and Z = 0.1 (metals)

Then  $\mu=1.4$  which gives us  $\ T_C=3x10^7\ K$ 

Internal energy that is available to be radiated is the kinetic energy of the nuclei (note that degenerate electron KE is not available). Each nucleus has energy =3/2 kT.

### White dwarf cooling

Total number of nuclei is:

$$\frac{M_{wd}}{AM_{H}}$$

(A is the number of nucleons per nucleus)

So the total energy is then: 
$$U = \frac{M_{WD}}{AM_{H}} \bullet \frac{3}{2} kT_{C}$$

A crude estimate of the cooling time is therefore  $\mathcal{T}_{cool} = \frac{U}{L_{wD}} = \frac{3}{2} \cdot \frac{M_{wD}k}{AM_{H}CT_{c}^{\frac{5}{2}}}$ 

Using A = 12 (carbon),  $T_C = 3x10^7$  and  $M_{wd} = 1 M_{sun}$  we get:  $T_{cool} \approx 170$  million years.

This is an underestimate as  $L_{\rm WD}$  decreases as energy is radiated away.

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# White dwarf cooling

A better cooling time estimate can be had as follows:

$$-\frac{dU}{dt} = L_{WD}$$
$$-\frac{d}{dt} \left( \frac{M_{WD}}{AM_{H}} \bullet \frac{3}{2} kT_{c} \right) = CT_{c}^{\frac{2}{2}}$$

Use boundary condition  $T_c = T_o$  at t=0

so

$$-\int_{T_o}^{t} T_c \frac{7}{2} dT_c = C \frac{2AM_H}{3M_{WD}k} \int_{0}^{t} dt$$
$$T_c(t) = T_o \left( 1 + \frac{5}{3} \frac{AM_H CTo^{\frac{5}{2}}}{M_{WD}k} t \right)^{\frac{-2}{5}}$$
$$= T_o \left( 1 + \frac{5}{2} \frac{t}{\tau_o} \right)^{\frac{-2}{5}}$$

Where  $\tau_{o}$  is the cooling timescale we estimated earlier

$$\tau_o = \tau_{cool}$$
 at  $t = t_o$ 

# White dwarf cooling

Using

$$L_{WD} = CT_{c}^{\frac{1}{2}}$$

$$L_{WD} = L_{o} \left( 1 + \frac{5}{3} \frac{AM_{H}C^{\frac{2}{7}}Lo^{\frac{5}{2}}}{M_{WD}k} t \right)^{\frac{-7}{5}}$$

$$L_{WD} = L_{o} \left( 1 + \frac{5}{2} \frac{t}{\tau_{o}} \right)^{\frac{-7}{5}}$$

Which is our required cooling curve for a white dwarf.



Figure 15.9 Theoretical cooling curves for 0.6  $M_{\odot}$  white-dwarf models. [The solid line is from Eq. (15.21), and the dashed line is from Winget et al., *Ap. J. Lett.*, 315, L77, 1987.]

### White Dwarfs: Radii from gravitational redshifts

- We can check our conclusions independently that the radii of White dwarfs are approximately 100 times smaller than the radius of the sun, R<sub>Ω</sub>.
- A photon escaping from the photosphere of the star loses energy as it climbs out of the gravitational potential of the star:  $\Delta F \quad \alpha \quad GM$
- Z is the redshift (change in photon wavelength), here a gravitational redshift, given by:

$$\frac{\Delta L}{E} = \frac{\varphi}{c^2} = \frac{GM}{Rc^2} = z$$
$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta \lambda}{\lambda}$$
$$\frac{\Delta \lambda}{\lambda} = \frac{GM}{Rc^2}$$

- We can measure the wavelength shift in the hydrogen lines of type DA stars and we find it is approximately 75 kilometres per second.
- · For many white dwarfs there is no velocity reference against which this can be checked.
- We can however apply this method when there is a white dwarf in a binary system and use the wavelength of the companion star for reference (provided, of course, that it is not also a star that is so compact to have a significant gravitational redshift).

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### **Neutron Stars**

- What happens if the mass of a degenerate object exceeds the Chandrasekhar mass of approximately 1.4 M  $_{\odot}$ ?
- When the degeneracy energy of the individual particles is greater than the mass difference between a proton and an neutron we find that protons and electrons combine by inverse β decay.
  - This occurs when:  $e^{-} + p > n + \nu \label{eq:massed}$   $(m_n m_p) \ c^2 \ \sim \ E_{deg} \ \sim 1.29 \ MeV$
- Just as with electrons, neutrons are subject to degeneracy pressure. For a given mass, the radius of the star is inversely proportional to the mass of the dominant degenerate particle  $M_{deg}$ . >>  $R_{NS} \sim R_{WD}(M_e/M_n) \sim 10^4 \text{ m}$
- As the mass of a neutron is approximately 2000 times that of the mass of an electron, the radius of a neutron star is approximately 2000 times smaller than that of a white dwarf held up by electron degeneracy pressure.

Mass proportional to  $1/(M_{Deg} \cdot R)^3$ 

 The radius of the sun, R<sub>☉</sub>, is approximately 7 x 10<sup>8</sup> metres, a white dwarf will then be approximately 2 x 10<sup>6</sup> metres, and a neutron star approximately 10<sup>4</sup> metres or about 6 miles.

#### **Neutron Stars**

- Neutron stars have remarkably extreme properties.
- You can work out that the neutrons are typically spaced by a distance of approximately 10<sup>-15</sup> metres, a distance which is approximately the size of an atomic nucleus.
- This means that the star has a nuclear density of the order of  $10^{14}$  kg m<sup>-3</sup>.
- The gravitational potential energy of the star which is, of course, GM<sup>2</sup>/R, is a substantial fraction of the rest mass energy, ~ 0.1 Mc<sup>2</sup>.
- This implies that general relativity is necessary when calculating accurately what happens on the surface of neutron stars and within their interiors.
- You can also work out that the mean velocity of the particles at the surface is ~ 0.6 c and that the potential energy of a particle at the surface is approximately 0.2 mc<sup>2</sup>.

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### **Neutron Stars and Black Holes**

- We can use exactly the same arguments which lead us from white dwarfs to neutron stars to conclude that as the mass increases neutron stars also shrink.
- Eventually, as the mass increases, we reach a point where the degeneracy pressure provided by neutrons cannot support a neutron star.
- This leads to the collapse into a black hole, where the escape velocity of photons from the surface is greater than the speed of light and therefore no radiation can be emitted from the surface of the star.
- At this stage our physical understanding of these objects becomes distinctly vague. We believe that for masses less than about three solar masses neutron stars can support themselves but beyond that they will collapse into a black hole.
- As with neutron stars we need general relativity to work out properly what is happening.
- Classically we would say that  $v^2 = 2GM/R$ . If we set v=c then we get the Schwartzschild radius equal to the size of a black hole given as:

 $R_{schw} = 2GM/c^2$ 

#### **Relative Sizes of Stars: from Normal Stars to Black Holes**



Fig. 1. Relative sizes of normal stars, white dwarfs, neutron stars and black holes having similar masses (we have taken  $1.4 M_{\odot}$ ). Note that while white dwarfs are much more compact than normal stars, they are not nearly as compact as neutron stars or black holes which, however, come rather close together.

**Discovery of pulsars** 

- The neutron was discovered in 1932 and the existence of neutron stars was hypothesised in 1934 and linked to what might happen in a supernova by Baade & Zwicky.
- This led to the prediction that there should be a magnetised neutron star in the centre of the Crab nebula in 1964 by Hoyle, Narlikar & Wheeler. The Crab nebula is a remnant of a supernova that exploded in 1054 AD, and was recorded by Chinese astronomers as a naked eye object for about 18 months.
- A graduate student, Jocelyn Bell, in the Radio Astronomy group of the Cavendish laboratory in Cambridge was trying to track down interference on a low frequency radio telescope by looking at the relatively rapid scintillations in radio intensity of distant radio sources caused by the passage of the radio waves through the solar wind.
- Good reference series in Science, vol 304, 23/4/04, special section on Pulsars and associated topics over pages 531 – 552.

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### **Discovery of Pulsars**

- Jocelyn Bell was looking through the hours of pen-chart plots. In particular she was tracking down interference which at the frequencies they were working at (81.5MHz) was rather serious.
- She noted interference that occurred at a repeatable sidereal time (rather than solar time) indicating that its source was extra-terrestrial.
- The first object was called LGM1 (for little green men). The subsequent discovery of another three around the sky made it very unlikely that this was intelligent life.
- She and Tony Hewish (who later received a Nobel prize with Martin Ryle) had discovered a periodic extra-terrestrial signal of 1.337 s at position: RA 19:19:36, DEC +21:47:16
- This was the first Pulsar (PSR J1921+2153).
- Not seen previously because most radio astronomy requires long integrations on faint sources rather than the high time resolution they were using for scintillation studies of the solar wind.



Pulsars are Neutron stars.

- The maximum rotation rates that a star can have is when the centripetal force at the surface is equal to the gravitational acceleration at that surface. Then  $\omega^2 R = GM/R^2$ , and the minimum rotation period,  $P_{min} = 2\pi/\omega$ , and  $P_{min} = 2\pi(R^3/GM)^{1/2}$ .
- and the minimum rotation period, P<sub>min</sub> = 2π/ω, and P<sub>min</sub> = 2π(R<sup>3</sup>/GM)<sup>1/2.</sup>
  This gives a minimum period of P<sub>min</sub> ~ 7 sec for a white dwarf but since R<sub>wd</sub>/R<sub>ns</sub> ~ 500, then P<sub>min</sub> ~ 5 x 10<sup>-4</sup> sec, which can be compared with one of the faster pulsars, the Crab pulsar, which has a period of 3.33 x 10<sup>-2</sup> sec or 30Hz.
- What do we expect for the actual rotation period of a neutron star? The precursor of a neutron star is a stellar core made out of <sup>56</sup>Fe. From earlier in this module, the mass and radius of a white dwarf (which is similar to the stellar core) are related by:
- And for a neutron star this is given by:
- Z = nuclear charge, A = nuclear mass so:
- And given that  $M_{wd} \sim M_{ns}$  we see that:
- Showing that the radii are very different:

 $R_{WD} \propto \frac{1}{m_e M_{WD}^{\frac{1}{3}}} \left(\frac{Z}{A} \cdot \frac{1}{m_p}\right)^{\frac{2}{3}}$  $R_{NS} \propto \frac{1}{M_{NS}^{\frac{1}{3}}} \left(\frac{1}{m_p}\right)^{\frac{8}{3}}$  $\frac{Z}{A} = \frac{26}{56} = 0.5 \text{ for Fe}$ 

$$\triangleright \frac{R_{WD}}{R_{NS}} \approx \frac{m_p}{m_e} \left(\frac{Z}{A}\right)^{\frac{5}{3}} \approx 500$$

### **Pulsar Properties**

- · We should expect pulsars to conserve angular momentum.
- The moment of inertia of a sphere is given by  $I = cMR^2$ , with a constant c which depends on the density as a function of radius. A uniform density implies that c = 2/5.
- Conserving ang. mom. gives us:

Assume M<sub>WD</sub> ~ M<sub>NS</sub>

$$I = cMR^{2}$$

$$I_{WD}.\omega_{WD} = I_{NS}.\omega_{NS}$$

$$M_{WD}.R^{2}{}_{WD}.\omega_{WD} = M_{NS}.R^{2}{}_{NS}.\omega_{NS}$$

$$>> \omega_{NS} = \omega_{WD} \left(\frac{R_{WD}}{R_{NS}}\right)^{2}$$
or  $P_{NS} = P_{WD} \left(\frac{R_{WD}}{R_{NS}}\right)^{2}$ 

$$>> P_{NS} \approx 5.10^{-3} \sec s$$

- White dwarfs are observed to have periods of about 1500 seconds and as such had been the fastest rotators until pulsars were found.
- We saw earlier that pulsars were about 500 times smaller than white dwarfs and this is why they rotate so rapidly.
- To put this in context, the sun is observed to rotate in about 25 days and a white dwarf in 1000-10,000 seconds (they are 100 times smaller than the sun typically).

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### **Pulsar Properties**

- So neutron stars are capable of producing periods of rotation of the right order and indeed the periods we observe are not surprising given what we already know about normal stars and white dwarfs, and of course the conservation of angular momentum.
- We still need explanations for:
- 1. The very wide range of rotation periods (these are observed to range from three seconds down to about one millisecond).
- 2. The fact that the pulses are only on for a very small fraction of the total period. Typically the duty cycle is about 5%.
- 3. The pulses are detectable over a very large range of frequency. They are difficult to detected at optical wavelengths but the Crab pulsar is seen at radio, optical, x-ray and gamma-ray wavelengths.
- This is a movie of the Crab pulsar taken in November on La Palma with our Lucky Imaging camera (CDM + Nick Law, IoA)



- The thermal emission implies that they are radiating as a black body with the characteristic temperature and wavelength but the pulse emission mechanism must be of a very different, non-thermal origin.
- What we will look at now it is the wide range of observational data that we have to pull together in order to understand what is going on in pulsars.



## Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer magnetic axis

#### **Pulsar Properties**

- The picture that we have is of a neutron star with a strong embedded magnetic field set at an angle to its rotation axis.
- The highly charged relativistic plasma near the surface can only be ejected parallel to the magnetic field.
- This produces a substantial beaming of radiation in the direction of the magnetic axis so that the distant observer only detects a pulse when the magnetic axis is pointing close to the line of sight.
- This also makes it clear that only a small proportion of pulsars will be visible. If the pulse duty cycle is only 5%, then only 5% of pulsars will be visible.



Graphics from Michael Kramer, Jodrell Bank.



### Pulsar Properties: where are they found?

• Early searches for a pulsating optical counterpart of the original pulsars were not successful. The first optical pulsations that were detected were from the Crab nebula.

31• Only five pulsars have been detected now at optical wavelengths but a great many are detected in the vicinity of supernova remnants.



140 ms zoom in on individual pulses Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

The individual radio pulses from pulsars are quite different from one another. They vary in shape and amplitude indicating a fairly complex geometry for the region that generates the radio pulses that we see.



Pulse longitude

The pulse shape obtained from adding together a few hundred pulses is quite stable and repeatable. Pulse drift implies cones are wandering but in a repeatable sort of way?



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## Pulsar Properties

• The pulse structure varies in quite a complicated way as a function of wavelength. The above picture shows a single pulse that has been detected by different radio telescopes around the world which were all synchronised with great accuracy. There is a tendency for the low frequency pulses to be broader

and more separated than they are at high frequencies.

Graphics from Michael Kramer, Jodrell Bank. Two more examples of pulse shape versus observed frequency showing that the width is narrower for higher radio frequencies.



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### **Pulsar Properties**

#### Pulsar Properties: the connection between radio and high-energy pulses

- The largest pulses detected at radio wavelengths appear to be synchronised with the high-energy (x-ray) emission.
- We also see structure on the timescale of nanoseconds. This is really quite astonishing, because the speed of light is only 30 centimetres per nanosecond and for us to be able to detect structure within pulses on nanosecond timescales implies that these events are localised to something that is at most a few metres across!





Vela

#### Pulsar Properties: the connection between radio and high-energy pulses

- The connection between radio and high-• energy pulses may also be related to the micro structure in normal pulses.
- The Pulsar emission and in particular the micro pulses are highly polarised, up to 100% elliptically polarised and often greater than 10% circular polarisation.
- The characteristic S-like swing of • polarisation PA (position angle) implies that this is an aspect of the viewing geometry.





### **Pulsar Properties**

### **The Life of Pulsars**

Bank.

Pulsar periods increase with time. The slowing down implies they are losing energy. We can estimate the energy loss rate as follows:

$$\begin{split} E_{rotation} &= \frac{1}{2} I \omega^2 \\ >> \frac{dE_{rot}}{dt} = I \omega \frac{d\omega}{dt} \\ (\omega \text{ from P}, \frac{d\omega}{dt} \text{ from } \frac{dP}{dt}) \\ \text{For Crab pulsar :} \\ P &= 3.33.10^{-2} \sec s \\ \frac{dP}{dt} &= 4.23.10^{-13} s s^{-1} \\ \text{Assume : } M &= 1.4 M_{sun} (M_{chand}) \\ >> R &= 1.2.10^4 \text{ m} \\ >> I &= 1.4.10^{38} \text{ kgm}^2 \\ \text{Energy Loss } > \text{Power} &= 6.10^{31} \end{split}$$



log Energy (keV)

Energy loss can be accounted for as Graphic from radiation from a spinning magnetic Michael Kramer, Jodrell Bank. dipole (see later).



Some pulsars show glitches in their spindown rate. These are due to "star-quakes" - small changes in the size of the outer crusts.

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### **The Life of Pulsars**

Pulsar periods when pulsar was formed: Braking J0537-6910 <14 ms

30337 0710.	<1 i mb	
B0531+21:	19 ms	rapidly
B1951+32:	27 ms	
B0540-69:	30 ms	
J0205+6449:	60 ms	
J1811-1925:	62 ms	
J1124-5916:	90 ms	
B0538+2817:	139 ms	

- As pulsars age they move to the right on this diagram until they reach the graveyard zone, where they are unable to sustain pulses (because the rotation energy and magnetic fields are declining).
- The pulses turn off for periods • of seconds up to hours, and some are seen more "off" than "on".
- Eventually they will vanish from sight. 42





### **The Life of Pulsars**

- When initially formed, pulsars are "born" with small rotation periods, on the order of 1 to 10 milliseconds, and large slow down rates, *P-dot*, on the order of 10<sup>-12</sup> seconds per second.
- They then evolve initially along lines of constant slow down rate.
- Eventually, their slow down rate decreases but their magnetic field also weakens, making their radio emission less powerful. When they are too faint to be detected, we say they have crossed the pulsar "death line" and have become invisible.
- It is believed that future millisecond pulsars enter the "graveyard" as members of binary systems. As the companion evolves, mass and angular momentum are transferred from the companion to the pulsar, spinning it up.



- Once "spun-up", the pulsar is "born again" as a millisecond pulsar.
- The only problem with this picture is that it would predict that all millisecond pulsars are members of binary systems. But they aren't.... However, one of them, PSR 1957+20, nicknamed the "Black Widow" appears to be evaporating its companion away. So perhaps all single millisecond pulsars are "black widows".

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### **Pulsars: Proper Motion**

- When the pulsar is formed, the supernova explosion is not necessarily symmetric and therefore the pulsar may be left with a significant velocity relative to the centre of the supernovae remnant.
- For example, in J0538+2816, the proper motion tells us that the age of the supernova is approximately 30,000 years.
- Similar measurements can be made at x-ray energies, and this has been done for the supernova remnant Puppis-A.



Kramer et al. 2003



**Fig. 3.** Chandra x-ray image of the SNR G292.0+1.8. The arrow marks the point source identified with the 135-ms pulsar PSR J1124-5916. The darker blue region surrounding the point source is x-ray emission with a hard power-law spectrum, believed to be a synchrotron emission from a relativistic pulsar wind. Credit: NASA/Chandra X-ray Center (CXC)/Rutgers University and (46).

### **Pulsars and Supernovae Remnants**

- This ROSAT x-ray image of the central part of the supernova remnant Puppis-A shows a bright point source with an x-ray luminosity more than 2000 times brighter than its optical luminosity (indicating it may well be a hot neutron star) about 6.1 arc minutes away from the centre of the supernova.
- The supernova remnant is about 3700 years old (measured from its expansion velocity) indicating that it is moving at between 500-1,000 km s<sup>-1</sup> relative to the centre of the remnant (from which it presumably originated).
- (From Petre et al, 1996, Ap J, <u>465</u>, L43).



### **Pulsars Masses**

- Oppenheimer & Volkov predicted that the mass of a neutron star would be approximately 1.4 solar masses. This is based on quantum mechanical calculations, and an exact value depends on the equation of state that is used to describe the relationship between the internal pressure and density of a neutron star as a function of mass.
- The equation of state is not well known for the high densities in neutron stars. By looking at binary pulsars (more about them later) we can measure the mass of a number of neutron stars.
- These observations show that the average mass is approximately 1.35 solar masses, very close to
- 47 the above theoretical prediction.



(Thorsett & Chakrabarty '99)

### **Pulsars: Internal Structure**

- Models of the internal structure of a pulsars are highly dependent on the equation of state used in the model.
- Pulsar timing observations suggest that we have the following structure:
- The overall diameter of a neutron star is probably about 20 kilometres (12 miles)
- The outer atmosphere is a superhot plasma.
- The outer crust has a structure that we get some information about from star quakes. This is a crystal lattice approximately 200 metres thick.
- There is an inner crust (here also we get information from star quakes) which is also crystal lattice approximately one kilometre thick.
- Then we have the outer core which is an atomic particle fluid with the density from  $10^{14} 10^{17}$  kg m<sup>-3</sup>.
- And then finally we have an inner core which is a solid chunk of subatomic particles.

### **Pulsars: Timing Information**

- It is possible to measure the period of a pulsar with astonishing accuracy. The period of B1937+21 is P = 0.0015578064924327±0.000000000000004 s, (1 microsecond in 100 years).
- It is a very complicated business to measure the arrival time of pulses with this sort of accuracy.
- Measurements have to be transformed to the barycentre of the solar system. The measurements
  have to take into account the relative position and motions of the pulsar, the telescope and the earth.
- They have to bear in mind that the pulsar is spinning down and therefore the exact instant at which the observation is made is critical. The timing measurements may be further complicated if the pulsar is part of a binary system.

There are likely to be relativistic effects and in young pulsars we see what are known as star quakes.



• From the timing information we can derive position, proper motion and parallax as well as lots of other details about the way these objects behave.

• These plots show a range of residuals found after the main slow-down is removed from the data.

Graphic from Michael Kramer, Jodrell Bank.

### **Pulsars as Magnetic dipoles**

- We know that white dwarfs have magnetic fields in the general range of B = 10 to 5 x 10<sup>4</sup> Tesla (1T=10<sup>4</sup>G). For comparison, the Earth's magnetic field is 30-60 microteslas.
- Both white dwarfs and neutron stars are made out of highly conducting material and therefore when the white dwarf collapses to create a neutron star the magnetic field which is frozen in the material of the star becomes compressed and therefore amplified.
- We have seen that the radius of a white dwarf is about 500 times bigger than that of a neutron star and therefore we should expect the magnetic field to be amplified during the collapse to a neutron star by a factor of perhaps 250,000. This will give a magnetic field of  $B = 10^9$  Tesla.

• We can see an amazing level of activity in some pulsars directly: this is the core of the Crab nebula, imaged by the HST at visible wavelengths (on the right), and by Chandra at X-ray wavelengths (on the left).



## **Pulsar Properties**



Graphic from Michael Kramer, Jodrell Bank.

- Pulses may be thought of therefore as an exotic form of cosmic lighthouse.
- 51• The pulse of emission is only detected when the beam points at the observer.



- This model shows most of the main features of what are now thought to be the mechanisms that are significant in pulsars.
- It is clear that these models need to be fairly complicated in order to account for the emission that one sees over such a wide range of wavelengths.

Graphic from Michael

Kramer, Jodrell

Bank.



#### **Pulsars as Magnetic Dipoles**

- With pulsars we are dealing with high rotation and high magnetic fields. The dipole model for • pulsars is very successful even though the details are still not understood fully.
- The magnetic field dominates for electrons at the surface of the neutron star. The ratio of the forces is given by: G '44' 1E  $CH (D^2)$

$$\frac{\text{Gravitational Force}}{\text{Magnetic Force}} = \frac{GMm/R^2}{e\omega RB/c}$$
  
\$\approx 10^{-12} (\$\omega\$ is angular velocity)

- This is true out to the radius where the co-rotation velocity equals the speed of light.
- Consider the pulsar as a spinning sphere with a magnetic field on its surface of **B**. The magnetic  $\Lambda \pi$ dipole moment M is given by:

$$M = \frac{m}{2\mu_o} .BR^3$$
[1]
$$\mu_o = \text{permeability of vacuum}$$

- The pulsar is spinning (therefore accelerating)
- The dipole emits radiation with a power P given by:

( $\alpha$  is the angle between the rotation axis and the magnetic field axis,  $\Omega$  is the angular velocity of the pulsar)

$$P = \frac{2}{3} \left( \frac{\mu_o}{4\pi} \right) \frac{1}{c^3} \left| \ddot{M} \right|^2$$
[2]  
$$\ddot{M} = M \sin \alpha \bullet \Omega^2$$
[3]

[2]

[4]

$$E_{rot} = \frac{1}{2}I\Omega^{2}$$
$$\frac{dE_{rot}}{dt} = I\Omega\dot{\Omega}$$

1 •

The total power emitted due to loss of rotational • energy is 53

### **Pulsars as Magnetic Dipoles**

We now equate the dipole power [2] with the rotational energy loss [4], and substitute for the magnetic dipole moment [1] and its second derivative [3].

This gives us the energy loss rate:

Forgetting about all the constants and assuming corotation out to the edge of the speed-of-light cylinder which has radius R<sub>c</sub>, and assuming this is still a simple dipole, we get:

The magnetic energy density  $\mathcal{E}_{c}$  is then given by:

and is proportional to the surface area of the cylinder The electrons arrive with a velocity v ~ c so that the power loss dE/dt is given by:

(Assuming that the relativistic electrons are stripped from the co-rotating cylinder at this critical radius and the energy is lost.) We do not know how this is 54done.

$$P = \frac{2}{3} \left(\frac{\mu_o}{4\pi}\right) \frac{1}{c^3} M^2 \Omega^4 \sin \alpha^2$$
  

$$= \frac{4\pi}{\mu_o} \cdot \frac{2}{3} \cdot \frac{B^2 R^6 \Omega^4 \sin \alpha^2}{c^3}$$
  

$$\frac{dE_{rot}}{dt} = I \Omega \dot{\Omega} = \frac{8\pi}{\mu_o} \cdot \frac{B^2 R^6 \Omega^4 \sin \alpha^2}{3c^3}$$
  
i.e we have  $\frac{dE_{rot}}{dt}$  in terms of  $B, R, and \Omega$   
 $M \propto B.R^3$   
 $B_c \approx B \left(\frac{R}{R_c}\right)^3$  (Field at  $R_c$ ) Dipole  
moment is  
 $\varepsilon_c \approx B_c^2 \mu_o^{-1}, area \propto R_c^2$  constant  
 $\frac{dE_{rot}}{dt} \propto \varepsilon_c R_c^2 c \propto \frac{B^2 R^6 \Omega^4}{\mu_o c^3}$ 

$$\therefore R_c \propto c/\Omega$$
, as before

### **Pulsars as Magnetic Dipoles**

• The slowdown rate is then:

$$\frac{dE_{rot}}{dt} = I\Omega\dot{\Omega} = (\text{constants}).B^2 R^6 \Omega^4$$

- Which we may express as a slowdown power law: (where k=constant, and n is a "braking index").
- Take natural logs of the power law and differentiate to give an expression for n.
- We need observations of d<sup>2</sup>Ω/dt<sup>2</sup> to determine this braking index.
- n=3 according to the above equation.
- Observations of the Crab pulsar gives n ~ 2.5

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## $\dot{\Omega} = -k\Omega^n$

 $n = \Omega \ddot{\Omega} / \dot{\Omega}^2$ 

### **Pulsars : pulse glitches**

- Pulsars exhibit well-defined spin down rates in the range  $dP/dt = 10^{-12} 10^{-17} \text{ ss}^{-1}$ .
- If you removed from the plot the long-term trend we see that about this trend for young pulsars there are discontinuous glitches where the pulsar speeds up.
- The amplitude of these glitches is typically ~ 1 part in 10<sup>8</sup>
- The Vela Pulsar shows glitches of one part in one million.
- The discontinuity is followed by a slow recovery in the rotation.
- During the glitch, angular momentum will be conserved, so that the speeding up that occurs must be caused by the radius shrinking.
- For  $R = 10^4$  m, the radius change must be  $\Delta R = 10^{-4}$  m.
- These are now identified as star quakes in the outer crystalline crust of the star, with average movements of only 0.1 mm.



### **Science with Pulsars**

- Because pulsars are such incredibly precise clocks they are very useful in many areas of fundamental physics.
- For example, in the binary pulsar PSR B1913+16, the timing measurements are so accurate that we can measure the orbit as it shrinks by about one centimetre per day.
- This measurement was the first to confirm the existence of gravitational wave radiation.
- This allows us to get some idea of what the background gravitational wave density is in the universe.



57 Kramer, Jodrell Bank.



- The interstellar medium is principally composed of hydrogen. It is at a temperature of T ~ 10,000K and so it is a plasma with free electrons.
- Radio waves interact with these electrons and the coupling between them is more efficient for long wavelengths which therefore travel more slowly through a plasma.
- This is effectively a refractive index of the interstellar medium which is wavelength dependent.
- For pulses of different frequency emitted at the same time the arrival time at the observer is going to depend on the mean electron density along the line of sight, the distance and v<sup>-2</sup>.
- For a fixed frequency interval the arrival time spread depends on the integral of electron density along the line of sight. This integral is called the dispersion measure or DM.
- A dense interstellar medium along a short distance gives an identical effect to a rarefied interstellar medium over a long distance.



### **Pulsars: Dispersion Measure**

• In fact the interstellar medium electron density is relatively constant and so the dispersion measure then gives us a **distance estimate**. Generally we also know quite a lot about the electron density in different directions within the galaxy and therefore we can refine these estimates more accurately.

• There are more than 2000 pulsars known and so we get a good distribution of objects throughout the galaxy.

• We also see that the distribution of pulsars close to the plane of the galaxy is rather similar to the distribution of young stars in the galaxy suggesting that these are the precursors of neutron stars.

• This makes sense because the neutron star is the end point in the evolution of a massive star which has a relatively short lifetime.



Fig. 3. Actual distances projected on the Galactic Plane  $d0_{\rm proj}$ , as function of actual distance to the Galactic Plane z0 of the pulsars in the sample obtained with Model B of Hartman et al. (1997) with the variations in the DM. The high z pulsars cover a large fraction of the pulsars with large actual distances

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### **Science with Pulsars**

- Another example is that we could measure the relativistic spin-orbit coupling in a binary pulsar known as Geodetic Precession.
- This was first predicted by Damour & Ruffini (1974).
- The precession rates expected in general relativity are (e.g. from Barker & O'Connell 1975, Börner et al. 1975):

$$\Omega^{p} = \left(\frac{2\pi}{P_{b}}\right)^{5/3} T_{\odot}^{2/3} \frac{m_{c}(4m_{p} + 3m_{c})}{2(m_{p} + m_{c})^{4/3}} \frac{1}{1 - e^{2}}, \ T_{\odot} = GM_{\odot}c^{-3}$$

• The predicted rate for B1913+16 is:

 $\Omega_{p} = 1.21 \text{ deg/year} = \text{once per} \sim 300 \text{ years}$ 

The rotation axis of the pulsar is precessing about the normal to the orbital plane. Compare this with the Earth orbiting the sun which precesses once every 26,000 years.

Graphics from Michael

Kramer, Jodrell Bank.

### **Science with Pulsars**

One of the effects of geodetic precession is that the pulse from one pulsar may not always be visible.
Because of this precession the line of sight will change and give us changes in the pulse shape and



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### **Science with Pulsars**

- This variation can be modelled by looking at the beam profile that we would expect and then seeing how it will be affected by geodetic precession.
- The model allows us to estimate the ratio of the intensities of the double pulses and their separation.



### Application of pulsar timing within globular clusters

- The globular cluster 47 Tuc contains • more than 20 millisecond pulsars with periods between three and six milliseconds.
- Their positions may be measured with sub-microarcsec accuracy via timing measurements.
- Most millisecond pulsars have a binary companion with typical orbital rotation periods of about 100 minutes.
- The proper motions can be measured with sub-milliarcsecs/year precision.
- The relative motion within the cluster has been detected and even accelerations in the cluster's potential well have been measured.
- The differential dispersion measures ٠ within 47Tuc have allowed the first direct measurement of the gas within a globular cluster.



Graphics Michael Kramer, Jodrell



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### Application of pulsars timing within globular clusters

The core of the globular cluster 47Tuc, with about 20 millisec pulsars within it, allows a detailed analysis of the gravitational potential with the cluster.

> Graphics from Michael Kramer, Jodrell Bank.



#### **Pulsars: exciting recent discoveries**

- To give some flavour of the extreme objects that have been discovered in recent years here are some examples:
- Long-period pulsars have been discovered with simple dipole magnetic fields: PSR J1847-0130: Period = 6.7 s, B-field = 10<sup>14</sup> G (McLaughlin et al. 2003)
- Binary pulsars with a super-massive companion: for example,PSR J1740-3052: has a period = 570 ms,  $P_b$ (binary period) = 230 days, orbital eccentricity e = 0.579, and a companion mass,  $M_c$ >11  $M_{sun}$ . Possible black hole. (Stairs et al. 2001, 2002)
- Binary pulsar with the longest orbital period: PSR J1638-4715: Period = 764 ms,  $P_b = 1744$  days, e = 0.81,  $M_c > 4.5 M_{sun}$ , and there is also evidence that this pulsar is an eclipsing binary (Lyne et al. ).
- Two double neutron star systems: PSR J1756-2251: Period = 28 ms, P<sub>b</sub> = 7.7 hrs, e = 0.18 (Faulkner et al.).
- Most relativistic system ever discovered: PSR J0737-3039: Period = 22 ms,  $P_b = 2.4$  hrs! The orbital diameter is [a sin(i)] = 0.03 AU, e = 0.09, and the precession rate is an astonishing  $\omega = 17$  deg/yr!
- This pulsar has a huge amount of geodetic precession: period only 75 yr!
- The binary is radiating so much gravitational wave energy that the two components will merge in ~85 Myr! (see GRBs later).
- All this will boost the LIGO detection rates for gravitational waves!

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### **Pulsars: exciting recent discoveries**

- First discovery of a double pulsar in 1993 by Hulse & Taylor (Nobel prize for this).
- The compact object orbiting the 23-millisecond pulsar PSR J0737-3039A, is another neutron star, and a detectable pulsar.
- The companion, PSR J0737-3039B, is rotating once every 2.8 seconds and orbits PSR J0737-3039A in only 2.4 hours.
- Both stars in this remarkable binary system have masses greater than that of our Sun, but are only 20km across and have an orbital separation which is less than the size of the Sun.
- Another unique aspect of the new system is the strong interaction between the radiation from the two stars.
- By chance, the orbit is seen nearly edge on to us, and the signal from one pulsar is eclipsed as it passes behind the other.
- This provides a unique opportunity to probe the physical conditions of a
  pulsar's magnetosphere, something that has never been possible before.

### Pulsars: exciting recent discoveries



- This movie shows the erratic variability of a jet of high-energy particles being ejected from the Vela pulsar, a rotating neutron star. The images cover a period of 2.5 years of observations.
- The behaviour of the jet shows firehose instabilities which are probably caused by an interaction between the jet and the surrounding medium through which it is moving.
- The jet is about 0.2 pc in length and shooting out ahead of the moving neutron star.
- The way the jet appears to maintain its width along its length implies that it is effectively
- confined by magnetic fields generated by the charge particles flowing along the axis of the jet.
   <sup>67</sup> The jet is bending and whipping about at about half the speed of light. The bright blobs are moving in the jet at similar speeds.

- A new method of measuring the mass of the neutron star has been demonstrated by American radio astronomers (Nature, 28/10/2010).
- They used the Shapiro delay, the delay that pulses experience because of the gravitational potential well surrounding a massive star (in this case a white dwarf).
- The pulses from a pulsar pass very close to the surface of the companion in this edge-on binary.
- This gives the mass of the white dwarf and hence that of the neutron star.
- At 1.97+/- 0.04 solar masses, it rules out almost all currently proposed hyperon or boson condensate equations of state,
- Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks.

### Weighing a Neutron Star



### Weighing a Neutron Star

- A total of 1752 data points were taken throughout a single orbit lasting 8.7 days with a timing resolution of approximately 1 µs.
- The top curve shows the best fit to the data with the solid line showing what is predicted from the Shapiro delay.
- The middle curve shows what happens with the best fit to the data but ignoring the Shapiro delay.
- The bottom plot shows the best fit residuals after removing the computed Shapiro delay.

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- An attempt to detect gravitational wave radiation by using an ultra sensitive laser interferometer: needs to detect a 10<sup>-18</sup> m length change over its 4 km arm length (uses multiple passes through each arm).
- Uses interferometers in several sites, allowing some directional
- 70 capability.

# LIGO: Laser Interferometer Gravitational Wave Observatory







LIGO: Laser Interferometer Gravitational Wave Observations.

<sup>71</sup> Credit:LIGO Scientific Collaboration and Virgo Collaboration/Georgia Tech/S. Ghonge & K. Jani