Introduction to Astrophysics: Module 2

Michaelmas Term, 2020: Prof Craig Mackay

Module 2:

- Magnitudes, HR diagrams, Distance determination,
- The Sun as a typical star (timescales, internal temperature and pressure, energy source).
- Stellar lifecycles
- Newtonian mechanics, Orbits, Tides
- Blackbody radiation, Planck's, Wein's and Stephan-Boltzman's laws
- What makes things glow: Continuum radiation mechanisms, emission line mechanisms, spectra.

Stellar Magnitudes

The brightness of a star is called its magnitude. The definition is

$$m_\lambda = -2.5 \log_{10} F + \text{constant}$$

The minus sign means fainter objects have numerically bigger values of magnitude.
F is the measured flux in some well defined wavelength interval (filter type).
The constant is determined by measuring F for a standard star (Vega $\equiv \alpha$ Lyr) and defining $m_\lambda = 0$.
The factor 2.5 in the equation was chosen to make this system approximate to the ancient system which had the brighest stars as first magnitude and the faintest stars visible to the naked eye as fifth magnitude.

Apparent magnitudes

Apparent magnitudes are written as a letter which refers to the passband.
The magnitudes in the Johnson Cousins system are
U, B, V, R, I, z, J, H, K, L
A measure of the colour of a star can be obtained by taking the difference between 2 magnitude measurements. e.g.
$U-B$ or $B-V$
These colours are of course flux ratios because magnitudes are logarithmic.
Dust along the line of sight will make a star seem fainter. Because short wavelengths are affected more the light is reddened. For a cluster of stars which are all at the same distance reddening can be measured using a colour - colour diagram.

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Johnson Cousins passbands

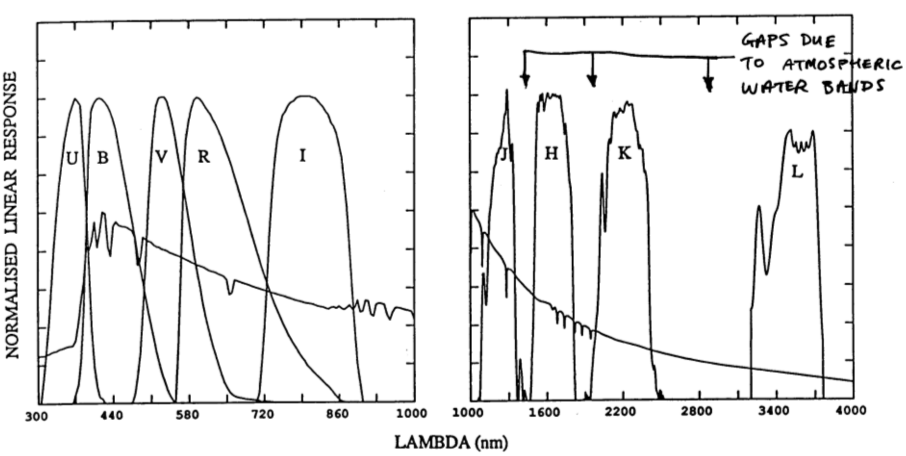

Figure 1. The passbands of the *UBVRIJHKL* system, plotted as functions of the wavelength in nm.

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Examples of Apparent Magnitudes

Sun	V=-26.7
Venus (at its brightest)	V=-4.5
Sirius (the brightest star in the sky)	V=-1.4
Vega (which defines the 0.0 of the system)	V=0.0
Faintest star visible to the naked eye on a city street	V=3.5
Brightest galaxy visible (Andromeda nebula,M31)	V=3.5
The faintest star visible to the naked eye from a dark site with an adapted eye	V=6
Brightest quasars known	V=12
Faintest object detected to date	V=29

(remember that 2.5 magnitudes is a factor of 10 so the Sun being 26.7 magnitudes bright is then more than 10^{10} brighter than the star Vega).

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Absolute Magnitudes

- The absolute magnitude is defined as the magnitude the star would have when observed from a distance of 10pc. It is therefore an intrinsic measurement of luminosity.
- Apparent magnitudes are denoted as m, U, B, V etc.
- Absolute magnitudes are denoted as capital letters with colour dependent subscripts such as M_U, M_B, M_V etc.

Examples:

Sun	Mv= 4.9
Faintest star	Mv = 18
Globular cluster	Mv = -7
Supernova at the peak of its brightness	Mv = -20
Andromeda (M31) galaxy	Mv = -20.5
A typical quasar	Mv = -25

Given the apparent magnitude of an object you need the distance to work out M (or alternately if you have M you can then work out the distance of an object)

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Extremely distant stars

Parallax

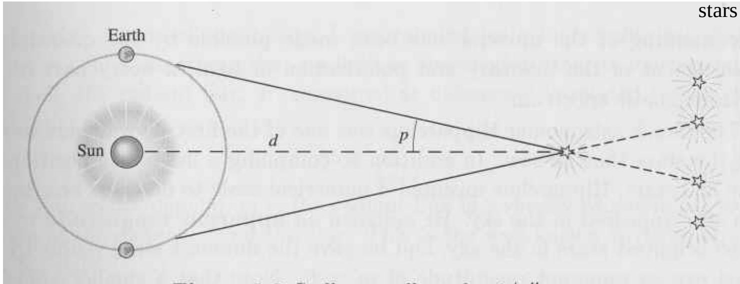

Figure 3.2 Stellar parallax: $d = 1/p''$ pc.

- Parallax is a fundamental technique of measuring distances that does not depend at all on the brightness of the target object.
- Biggest stellar parallax is 0.77 arcsec for Proxima Centauri.
- Can measure down to 20 milli-arcsec using ground based telescopes, i.e. out to ~50 parsecs.
- Proper motion makes ellipse appear as a helix
- Hipparcos satellite measured parallaxes down to 1-2 milli-arcsec (500 –1000 parsec)
- The European GAIA satellite is able to measure parallax to about 10 micro-arcseconds for over 50 million objects. This will allow precision measurement of distances of stars that are at the distance of the centre of our own Galaxy for the first time. It is due to launch in 2011.

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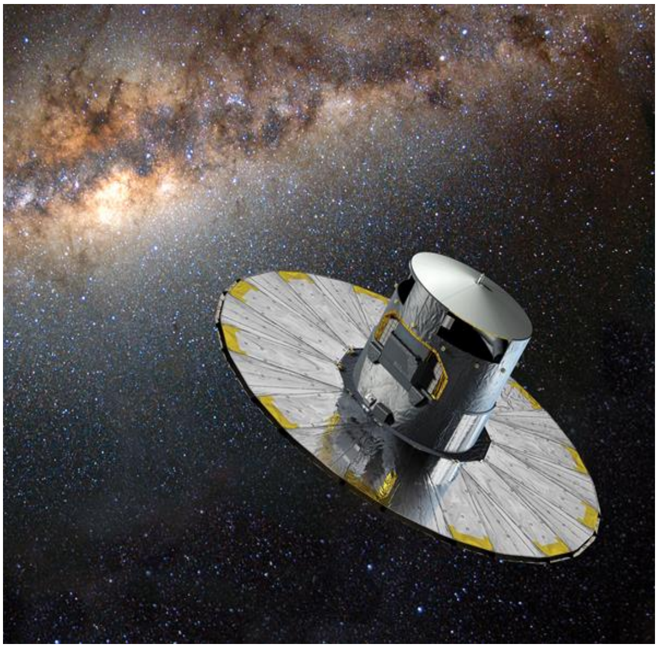

Gaia: 1 billion stars in three dimensions

- The European GAIA satellite is able to measure parallax to about 10 micro-arcseconds for over 50 million objects.
- Allow precision measurement of distances of stars that are at the distance of the centre of our own Galaxy for the first time.
- The satellite is located at L2, a stable point about 1.5 million km from Earth.
- The satellite has no moving parts so is very reliable and has not got much to go wrong!

- Generates a vast amount of data and needs a large team to process the results for the community.

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Distance Modulus

$$\text{Distance modulus} = m - M$$
$$m - M = -2.5\log F_d + 2.5\log F_{10}$$
$$= -2.5\log\frac{F_d}{F_{10}}$$

where F is the flux (energy/unit time in passband). Using the inverse square law, $F \propto 1/d^2$, where d is the distance, we have

$$\frac{F_d}{F_{10}} = \left(\frac{10}{d}\right)^2$$

$\therefore$
$$m - M = -2.5\log\left(\frac{10}{d}\right)^2$$

$$m - M = 5\log d - 5$$

Solving the last equation for d gives

$$d = 10^{(m-M+5)/5}$$

If we think we can guess M with some confidence then we can measure m and determine d. A distance determined in this way is called a photometric distance.

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How do we estimate absolute mag *M*

- For stars observed by the astrometric satellite Gaia we know d so we can get $M \rightarrow$ standard candles
- More standard candles can be established by association with the Gaia ones.
- Some standard candles can be several steps removed from a geometric distance determination.

Examples of Standard Candles

- Nearby stars of known type
- Main Sequence in the HR diagram
- Cepheid Variable stars
- RR Lyrae Variable stars
- Supernovae

- The problem is that none of these genuinely and totally are standard candles: they are only an approximation to one and therefore we have to be very careful that we understand the limitations implicit in using each of these so-called standard candles

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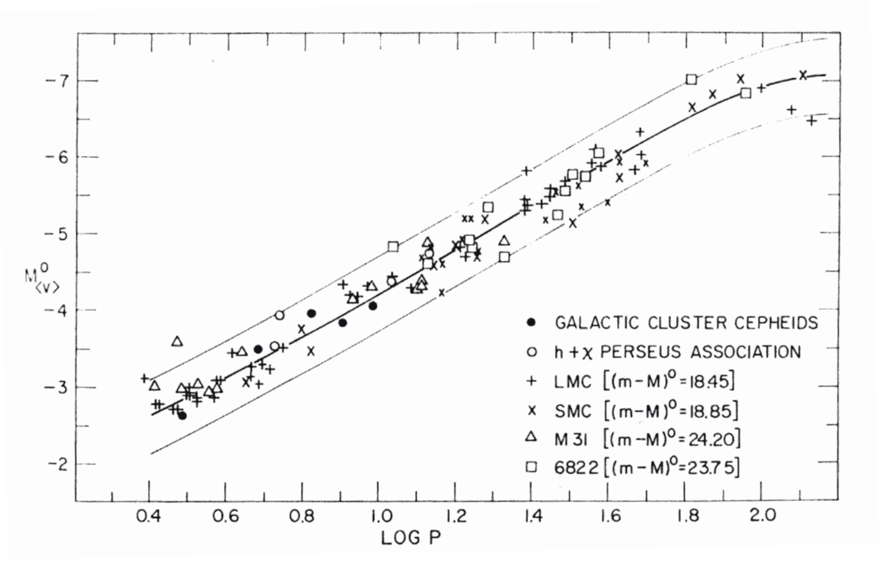
Period Luminosity Relation for Cepheid Variables



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Hubble's Law

Hubble plotted recession velocity (measured from spectra) against distance (estimated using standard candles) for nearby galaxies. He found that recession velocity is proportional to distance

$$v = H_0 d \qquad (H_0 \text{ is the Hubble constant})$$

$$d = v / H_0 = cz / H_0 \qquad (z \text{ is the redshift})$$

He estimated $H_0 = 500$ kms/sec/Mpc which is too large by about a factor of 6. (He underestimated the distances). For large redshifts the proper relativistic form should be used. This depends on the geometry of the universe. For a flat universe

$$d(t_0) = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

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Hubble's Law

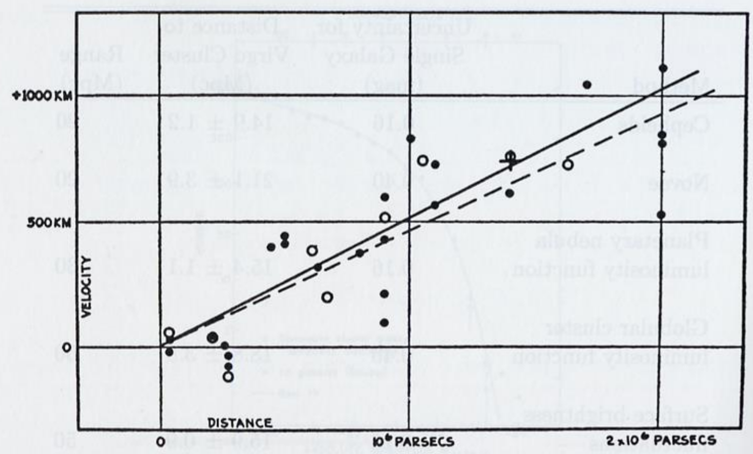
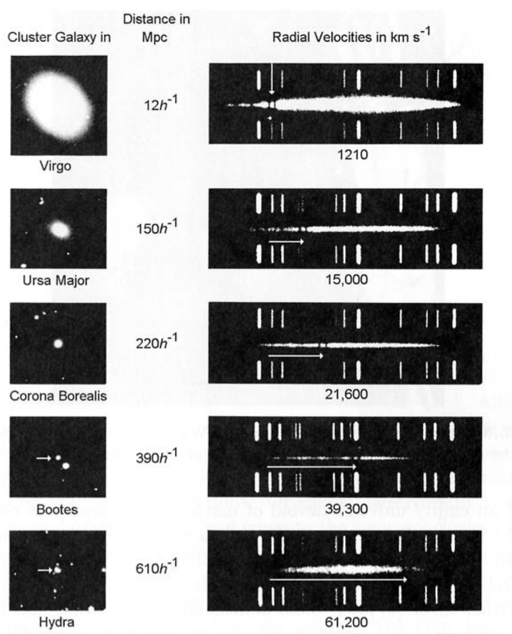


Figure 25.6 Hubble's 1936 velocity–distance relation. The two lines use different corrections for the Sun's motion. (Figure from Hubble, *Realm of the Nebulae*, Yale University Press, New Haven, CT, © 1936.)

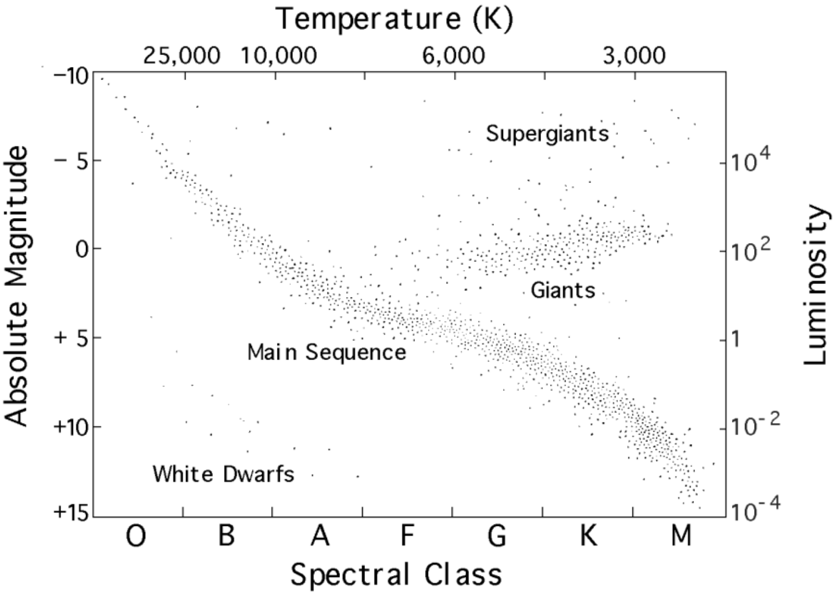


Hubble's Law

h=H_0/100

This means that the distances quoted would be correct if the Hubble constant was 100 km/sec/MPc

Hertzsprung-Russell (HR) diagram



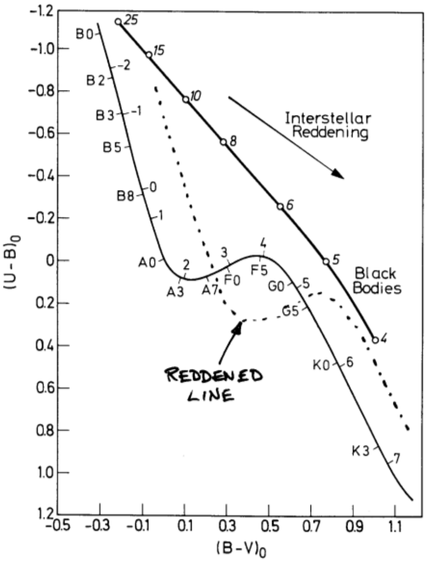
Hertzsprung-Russell (HR) diagram

- This is a plot of stellar surface temperature against stellar luminosity.
- Temperature is plotted on the x axis with the coolest values to the right. Both axes are logarithmic.
- Values may come either from observations or theory.
- A point represents a single star.
- An *evolutionary track* represents the way a single star evolves with time.
- An *isochrone* represents a sequence of stellar models of different masses but all with the same age (and initial composition).

How do we measure temperature?

- From spectral type (class) – relative line strengths strongly dependant on temperature.
- From apparent magnitudes – colours such as U-B and B-V have a well defined dependence on temperature. These quantities are essentially flux ratios because magnitudes are logarithmic.

Temperature is not so easy to measure: colour is not enough.



Colour also depends on inter-stellar reddening by dust along line-of-sight
This means that any temperature measurement has to be corrected for reddening.

Figure 15.5. Two-color diagram for the main sequence. The reddening-free color indices $(U\text{-}B)_0$ and $(B\text{-}V)_0$ are after H. L. Johnson, W. W. Morgan and others. The MK spectral types and absolute magnitudes of the stars are written along the main sequence. The almost straight line above the main sequence represents black radiators, the numbers are $T_{eff} \times 10^{-3}$ K. Interstellar reddening shifts the position of a star in the diagram parallel to the line in the upper right, which is drawn in particular for O stars.

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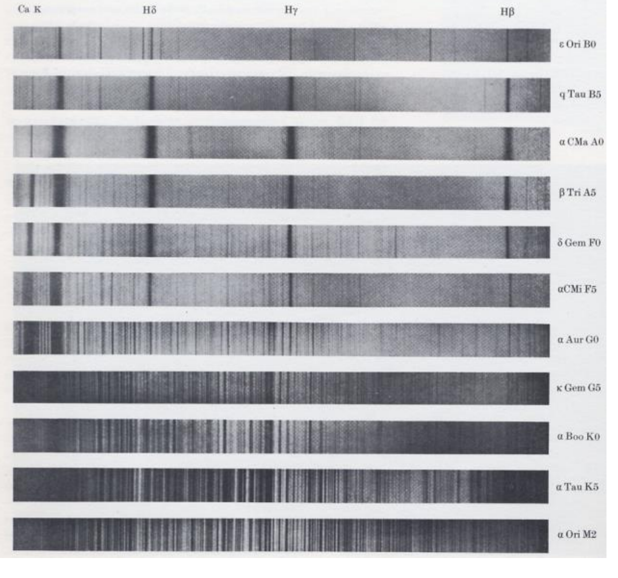
Spectral classification of main sequence stars

The spectra shown here cover the wavelength range from 390-490 nm



Hot

Cool

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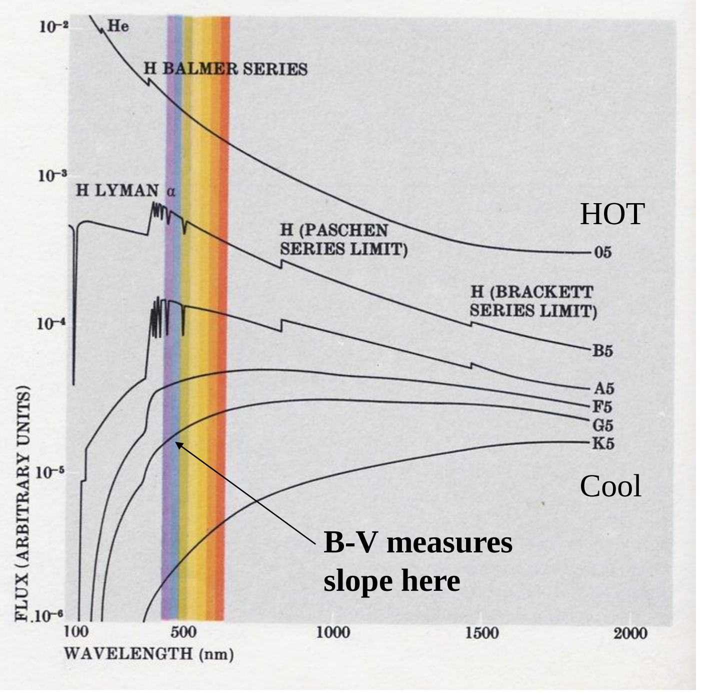

The colours of the stars

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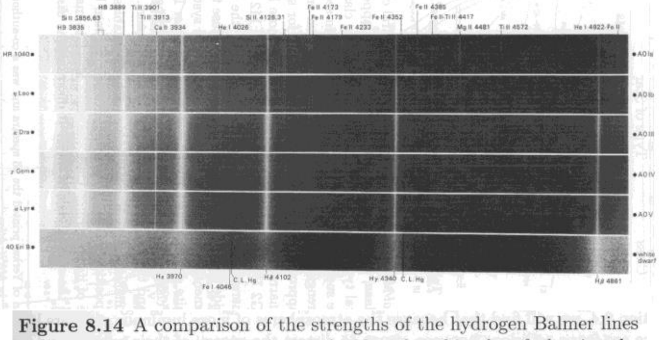
- **How do we measure luminosity?**

Absolute magnitude is a measure of luminosity and can be estimated from a star's spectrum.
Can use relative luminosities (i.e. apparent magnitudes) if all the stars plotted are at the same distance (i.e. in a star cluster).
This gives us the concept of *Luminosity classes* - AO stars shown below



Big

Small

Figure 8.14 A comparison of the strengths of the hydrogen Balmer lines in types A0 Ia, A0 Ib, A0 III, A0 IV, A0 V, and a white dwarf, showing the narrower lines found in supergiants. (Figure from Yamashita, Nariai, and Norimoto, *An Atlas of Representative Stellar Spectra*, University of Tokyo Press, Tokyo, 1978.)

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The Main Sequence and lines of constant stellar diameter,R

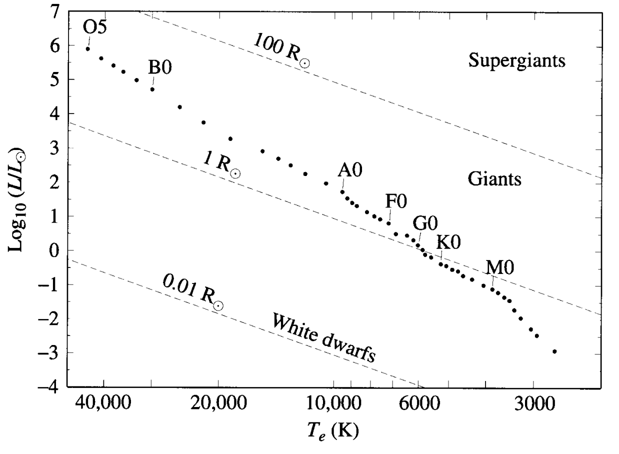

Figure 8.13 Lines of constant radius on the H–R diagram.

$$L = \text{const.} R^2 T^4$$

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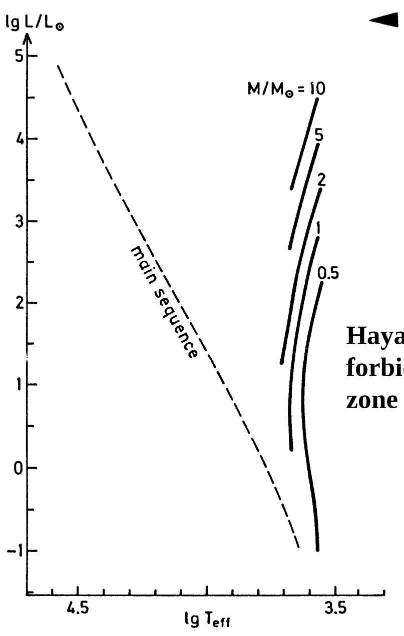

Hayashi forbidden zone

Hayashi lines for 0.5 to 10 solar masses

- The Hayashi tracks show the way that stars approach the main sequence where they will be have achieved pressure balance between gravity and the nuclear energy generation inside the star.

- To the right of these lines stars are unstable (they collapse, no pressure-gravity balance).

- A point on the Hayashi line corresponds to a fully-convective star (energy transport is via convection).
- For a given stellar mass there are many possible fully-convective stellar models.
- On the cool side of the line the star is unstable, has insufficient pressure support, hence region is called the Hayashi forbidden zone.
- These concepts are important for explaining stellar evolution.

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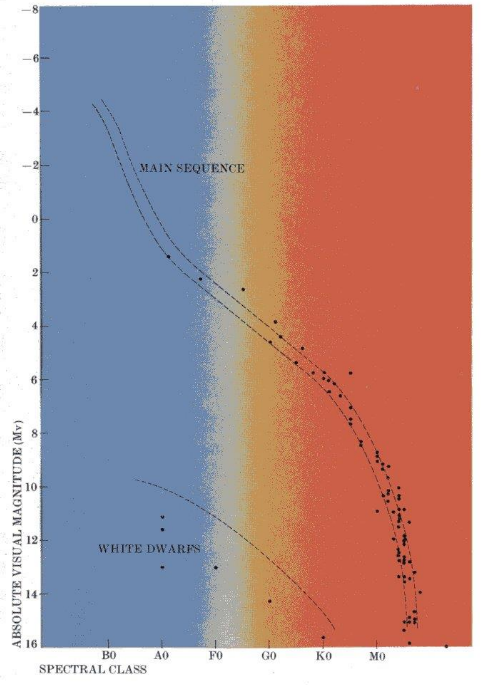

HR diagram for the nearest stars

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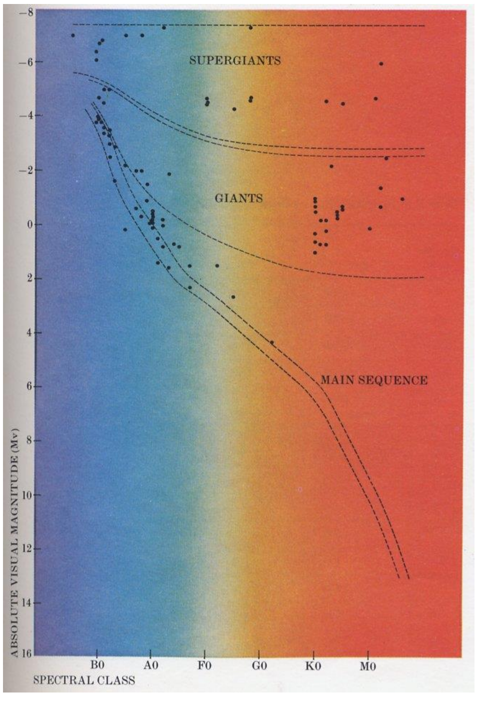

HR diagram for the brightest stars

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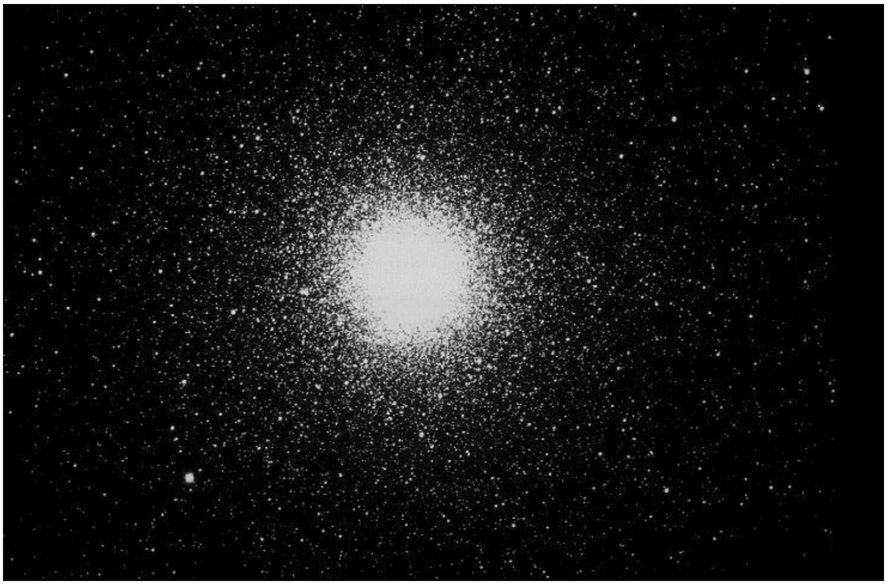
Globular cluster - M3



Colour-magnitude diagram globular cluster M3

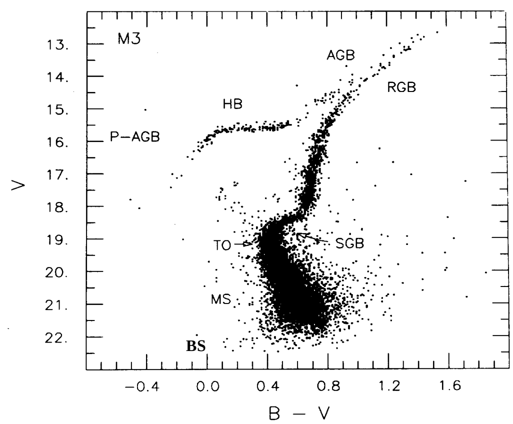


Figure 1. The color-magnitude diagram (visual brightness V versus color index $B - V$) of M3 shows the large variety of stars present in a globular star cluster. In this case the density of data points in the diagram is roughly proportional to the true number of the corresponding kinds of stars, and therefore indicates how long stars exist in the respective phases of evolution. Stars spend most of their time on the main sequence (MS), burning hydrogen in their cores. The turn-off point (TO) marks the end of the MS phase and thus is an indication of the age of the cluster. The TO defines the start of the subgiant branch (SGB) phase with shell hydrogen burning, in which the star increases in brightness and evolves to the red giant branch (RGB). Here the star loses mass and eventually becomes a horizontal branch (HB) star. The final evolution of the star may lead up to the asymptotic giant branch (AGB) and into the very hot post-AGB phase. *(From A. Renzini and F. Fusi Pecci. Reproduced, with permission, from* Annual Review of Astronomy and Astrophysics **26**, *©1988 by Annual Reviews Inc.)*

- Stars in the globular cluster may be assumed to be all of the same age and of the same chemical composition. This allows us to look at the rate of evolution of the star as a function of its mass independent of other considerations.

HR diagram terminology

MS – main sequence	ZAMS – zero-age main sequence	RGB – red giant branch
SGB – sub-giant branch	AGB - asymptotic giant branch	P-AGB - post-asymptotic giant branch
HB – horizontal branch	TO – turn off (main sequence turn off)	BS – blue stragglers

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HR diagram for an open cluster

Open clusters also consist of stars of the same age and chemical composition but their stars are, in general, significantly younger than those in globular clusters.

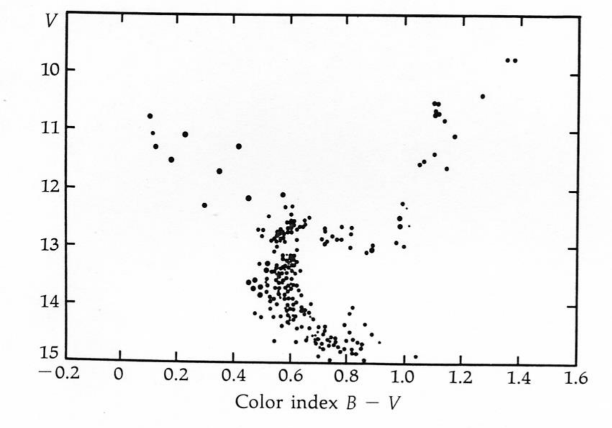

Figure 1.7 The Hertzsprung–Russell diagram for the old open star cluster M67 in the constellation Cancer (H. L. Johnson and A. R. Sandage).

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Pleiades – a cluster of recently formed stars (~100 Myrs old) a few pc in diameter and ~ 100 solar masses.

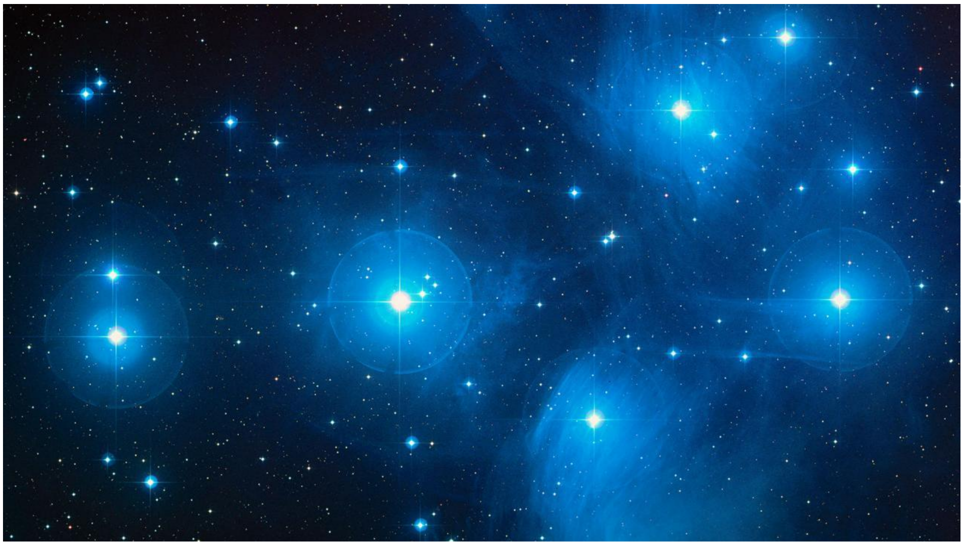
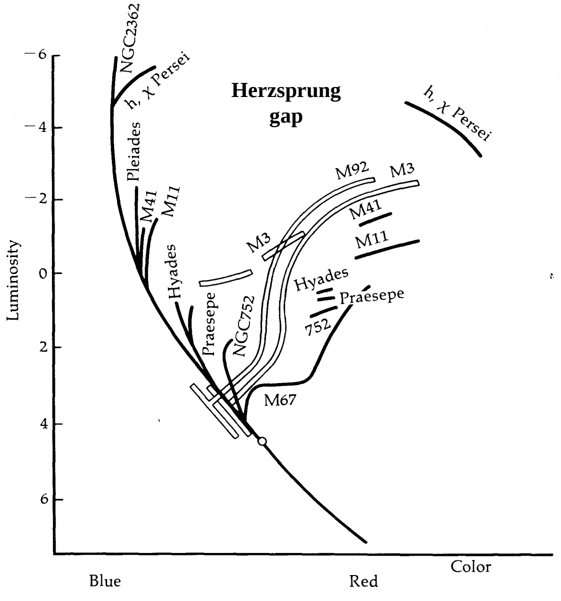


Figure 12.8 Composite Hertzsprung–Russell diagram for various star clusters (H. L. Johnson and A. R. Sandage).



HR diagrams for various star clusters

The Sun – a typical star - Free Fall Collapse

- Gravity holds the sun together but what would happen if only gravity was important?

- We can work out the free fall collapse time for the sun (any spherical gas cloud in fact).
- Assume that the sun is spherical and has uniform density, mass M, and radius R.
- The mass interior to r is:
  this acts as a mass at r=0 (Newton) $$M(r) = \int_0^r \rho(r') 4\Pi r'^2 \, dr'$$

  the acceleration is then: $$g(r) = \frac{GM(r)}{r^2}$$
  (G= gravitational constant)
- We assume initially that the shell of material is at rest and so the kinetic energy KE=0
- During the collapse the potential energy is transformed into kinetic energy.
- In order to conserve energy: $$\frac{1}{2}\left[\frac{dr}{dt}\right]^2 = \frac{Gm_o}{r} - \frac{Gm_o}{r_o}$$

- to evaluate the integral $$t_{ff} = \int_0^t \frac{dt}{dr} dr = -\int_{r_o}^0 \left[\frac{2Gm_o}{r} - \frac{2Gm_o}{r_o}\right]^{-\frac{1}{2}} dr$$

  Let $r=r_0x$ $$t_{ff} = \left(\frac{r_o^3}{2Gm_o}\right) \int_0^1 \left(\frac{x}{1-x}\right)^{\frac{1}{2}} dx$$
- substitute $x = \sin^2 \Theta$
- note that the mean density ρ, is: $\rho \approx \frac{m_o}{r_o^3}$ $\rho=3m_0/4\pi r_0^3$

Free Fall Collapse

- and therefore we find for the sun that:
- therefore something holds the sun up

$$t_{ff} = \left(\frac{3\Pi}{32G\rho}\right)^{\frac{1}{2}}$$

$$t_{ff} \approx 1750s$$

- Let us work this out again in a simpler way!

R=radius of Sun
V_{ff}= final velocity

$$t_{ff} \approx \frac{R}{V_{ff}}$$

- The final KE = the initial PE so:

$$V^2{}_{ff} = \frac{2GM}{R}$$

$$t_{ff} \approx \left(\frac{3}{8\Pi G\rho}\right)^{\frac{1}{2}}$$

- Quite close to what we originally got when we worked it out properly!
- We deduce that the collapse is very rapid (indicating the importance of gravity on large scales), and that real objects collapse inside first because the density increases dramatically as the radius vanishes.
- Also, that density is a key parameter.
- However it is quite clear that something else must be holding the Sun up!

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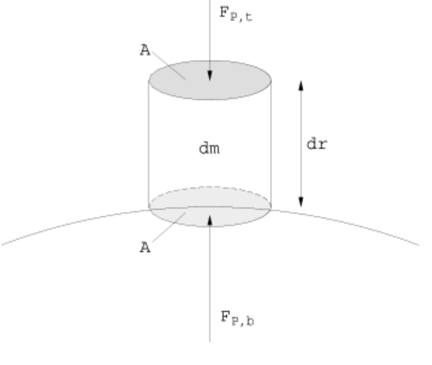
Hydrostatic equilibrium

- We know (because we are here) that the sun is stable over a very long timescales.
- This implies that gravity is balanced by an internal pressure. We consider the forces acting on a small volume element in the atmosphere of the sun.
- Radius r > r+ Δr, cross-section ΔA, and the volume Δr.ΔA, Mass ρ(r).Δr.ΔA



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Hydrostatic equilibrium

- force on volume element due to the radial pressure gradient is then given by: $\left[P(r)+\frac{dP}{dr}\Delta r-P(r)\right]\Delta A=\frac{dP}{dr}\Delta r\Delta A$

  $\Delta r\Delta A=\frac{\Delta M}{\rho(r)}$
- if we include gravity, we get: $-\frac{d^2r}{dt^2}=\frac{GM(r)}{r^2}+\frac{1}{\rho(r)}\frac{dP}{dr}$
- as it is in equilibrium we can set: $\frac{d^2r}{dt^2}=0$
- implies that the pressure gradient is: $\frac{dP}{dr}=\frac{-GM(r)\rho(r)}{r^2}$
- the system will be in hydrostatic equilibrium if this is true for all radii.

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Kelvin-Helmholtz Timescale

- The Kelvin-Helmholtz timescale, t_{KH} is a thermal timescale while the Free-Fall timescale, t_{FF} is the dynamical timescale.
- The Gravitational binding energy of a system is: $E_{Gr}\approx-\frac{GM^2}{R}$
- This is promising because $M_\odot$ is large.
- Assume $L_\odot$ is ~ constant. Then we have that: $t_{KH}\approx\frac{GM_{sun}^2}{R_{sun}L_{sun}}$
- For the Sun, t_{KH} ~ $3\text{x}10^7$ years, but the age of the Earth is ~ $4.5\text{x}10^9$ years.
- So we deduce it is not the conversion of gravitational energy in to thermal energy that keeps the Sun powered.

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The Virial Theorem

- For a self-gravitating system of particles which is in equilibrium it can be shown that

2KE = -PE

- This is the ***virial theorem.***

- KE is the kinetic energy of the particles and PE is the gravitational potential energy of the particles.
- "In equilibrium" means the system is neither expanding or collapsing.
- The particles could be atomic particles (system is a star) or stars (system is a star cluster or galaxy) or galaxies (system is a cluster of galaxies).

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Temperature and Pressure in the Sun

- We are working with a system that is in hydrostatic equilibrium, and we know from the Virial theorem that 2x Kinetic Energy = - Potential Energy, or 2KE ~ -PE.
- Thus 2 V <P> ~ GM^2/R, where V = volume, and P = pressure
- So <P> is given by: $\langle P \rangle \cong \frac{3GM^2}{8\Pi R^4}$ - [1]

  A very high pressure! (1 atmosphere = 10^5 Pa) $\approx 10^{14}\,Pa$

- For a perfect gas, the pressure is: $\langle P \rangle = nkT = \frac{\langle \rho \rangle}{\overline{m}} kT$ - [2]

- Using $<\rho>= M \Big/ \left[\frac{4}{3}\pi R^3\right]$

- and equating [1] and [2] gives

$$kT \approx \frac{GM_{sun}\overline{m}}{2R_{sun}}$$

$$T \approx 5 \bullet 10^6 K$$

- These temperatures are high enough to sustain nuclear fusion.

- This gives us a new power source with the capacity to deliver as much Energy as $E = mc^2$.
- With 100% conversion efficiency we would get a lifetime for the sun of 10^{13} years. Even with a 1% efficiency we would be able to account for the entire energy demand for the lifetime of the sun for 100 billion years which is still more than the age of the Universe.

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What supplies the energy for the Sun and the stars?

- Sun is ~4.5 billion years old.
- If its total gravitational energy is used up at its present luminosity it will last for ~30 million years.
- If the Sun's mass was made of dynamite the available energy would only last 10,000 years.
- Virial theorem tells us that as the Sun contracts, gravitational energy is converted into thermal and it gets hotter – so energy source can't be the thermal energy. (Note: this implies a self gravitating object like a star has negative specific heat).
- If the sun is made entirely of H and all this is turned to He by nuclear burning then lifetime at present luminosity is 100 billion years – must be nuclear.

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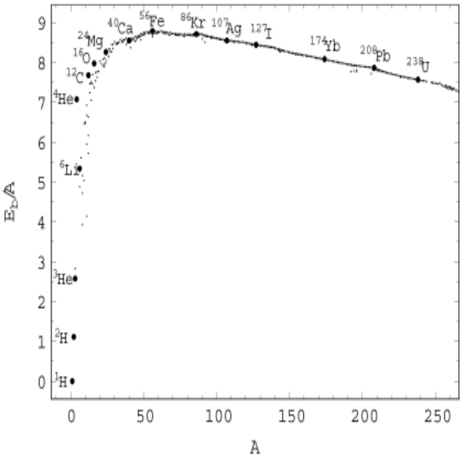
Nuclear Energy Generation

- The mass of the atomic nucleus is less than the mass of the nucleons (protons and neutrons) that make it up.
- The binding energy per nucleon for elements in the periodic table shows a maximum at iron (^{56}Fe), the most tightly bound nucleus.
- Combining light nuclei to make heavier nuclei up to ^{56}Fe produces energy by a process known as nuclear fusion.
- Combining nuclei to make even heavier nuclei requires energy to be added to the process and fusion is not possible.
- In the mass regime above ^{56}Fe it is possible to liberate energy by splitting nuclei into smaller components (fission) but the energy liberated is small compare to that available from fusion of hydrogen up to iron.
- This is essentially why H-bombs (fusion bombs) are so much more powerful than A-bombs (fission bombs).



- This figure shows the binding energy per nucleon in MeV, i.e. the amount of energy that is available from nuclear fusion that involves converting one element into another more massive one.

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Nuclear Energy Generation

- The amount of energy that is released by fusion is enormous. $^1H \rightarrow {}^4He$ gives $6.4 \times 10^{14} Jkg^{-1}$ and $^1H \rightarrow {}^{56}Fe$ gives 7.6×10^{14} Jkg^{-1}.
- Approximately 1% of the rest mass can be converted to energy by fusing hydrogen to helium. Nuclear fusion processes require high temperatures and high pressures in order for them to proceed.
- The central temperature inside the Sun is $T \sim 2 \times 10^7$ K, typical of low mass stars. Atoms are stripped of electrons and where the temperature and pressure are high the particles may be treated as a perfect gas with $P=nkT$.
- There are two principal routes for fusing hydrogen to helium. The first is for low mass stars, $<1.5\ M_\odot$, the **proton-proton chain (PP chain):**

$$^1H + {}^1H \leftrightarrow {}^2D + e^+ + \upsilon$$
$$^2D + {}^1H \leftrightarrow {}^3He + \gamma$$
$$^3He + {}^3He \leftrightarrow {}^4He + {}^1H + {}^1H$$

- This process takes six protons and generates a helium nucleus and two protons so four protons are used to give one helium nucleus. Each of the first two of the three stages occurs twice for one reaction of the third stage.

Nuclear Energy Generation

- The second route is the **carbon-nitrogen-oxygen cycle (CNO cycle).**
- This is relevant to higher mass stars, with masses $> 1.5\ M_\odot$.
- This uses C, N and O to act as catalysts to help the fusion of hydrogen to helium.

$$^{12}C + {}^1H \leftrightarrow {}^{13}N + \gamma$$

^{13}N decays to $^{13}C + \upsilon + e^+$

$$^{13}C + {}^1H \leftrightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^1H \leftrightarrow {}^{15}O + \gamma$$

^{15}O decays to $^{15}N + \upsilon + e^+$

$$^{15}N + {}^1H \leftrightarrow {}^{12}C + {}^4He$$

Energy generation from hydrogen burning

PP chain $\varepsilon \propto T^4$
CN cycle $\varepsilon \propto T^{20}$

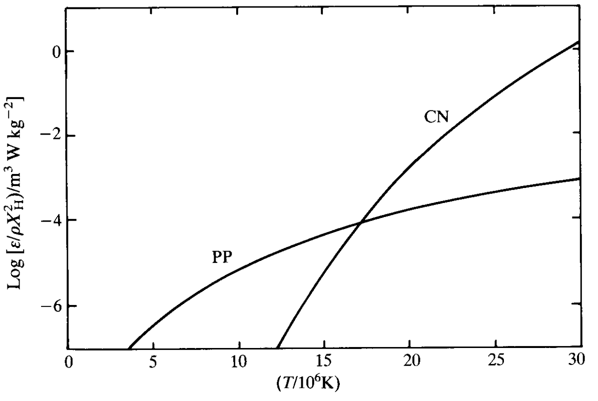


Fig. 39. Rate of energy release from hydrogen burning as a function of temperature for the two reaction chains.

Stellar Evolution

- Stars spend most of their life burning hydrogen into helium.
- Hydrogen is the main constituent and it is the transformation of hydrogen into helium which liberates most of the energy available by nucleosynthesis in stars.
- The nuclear fusion processes are extremely sensitive to temperature.
- The higher mass stars have very much higher central temperatures and therefore much higher luminosities.
- We find that $L \sim M^3$ approximately for H-core-burning stars (i.e. stars on the main sequence).
- Provided stars are sufficiently massive ($M > \sim 8M_{sun}$) they will be hot enough such that fusion of carbon, oxygen etc. all the way up to iron is possible.
- Fusion of He takes place via the triple α process. This three-particle process is necessary because it overcomes the the lack of stable nuclei with progressively increasing binding energy between helium and carbon.
- Here we have:

$$^4He + {}^4He \leftrightarrow {}^8Be + \gamma - 91.78\ KeV$$
$$^8Be + {}^4He \rightarrow {}^{12}C + \gamma + 7.367\ MeV$$

Nuclear Burning

Star with mass ~ 25M_{sun}

	Reaction	min temp	timescale
•	H $\rightarrow$ He	Log(T)=6.7	70 Myr
•	He $\rightarrow$ C	Log(T)=8.0	500 kyr
•	C $\rightarrow$ O	Log(T)=8.8	600 yr
•	O $\rightarrow$ Si	Log(T)=9.0	6 months
•	Si $\rightarrow$ Fe	Log(T)=9.6	1 day

Stellar Evolution

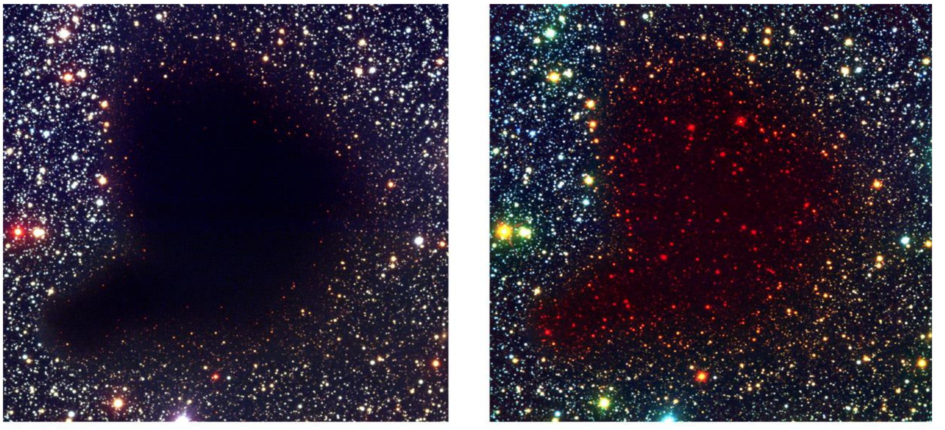
- Once carbon has been synthesised then the progression to iron goes via:

$$^{12}C + {}^{4}He \rightarrow {}^{16}O + \gamma$$
$$^{16}O + {}^{4}He \rightarrow {}^{20}Ne + \gamma$$
$$^{20}Ne + {}^{4}He \leftrightarrow {}^{24}Mg + \gamma$$

(this process continues right up to ^{56}Fe)

- This process produces little energy compared to the fusion of hydrogen to helium, and therefore the lifetime of this stage in the evolution of the star will be very short because it simply is not able to produce enough energy to maintain a stable configuration for any length of time.

Bok Globule



B, V, I B, I, K

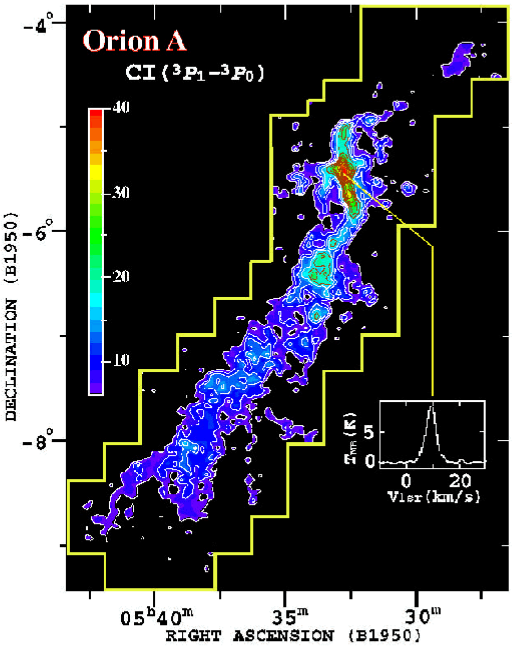
Pre-Collapse Black Cloud B68 (comparison)

About 0.1 pc diameter and a few solar masses.

Multi-colour images. B=0.43um, V=0.55um, I=0.80um, K=2.2um 45

Giant Molecular Cloud in Orion

- This is a radio map in the middle of the Orion nebula in a molecular line.
- This region is extremely cold, only a few degrees above absolute zero.
- ~10^6 solar masses, and ~50 pc in size.



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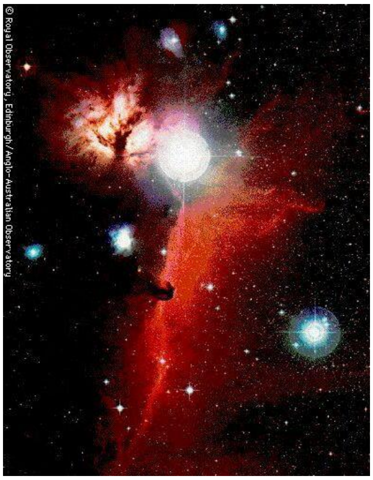
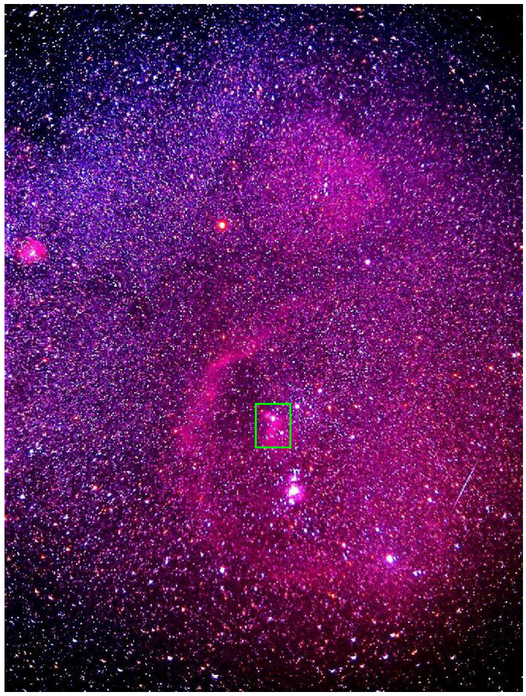
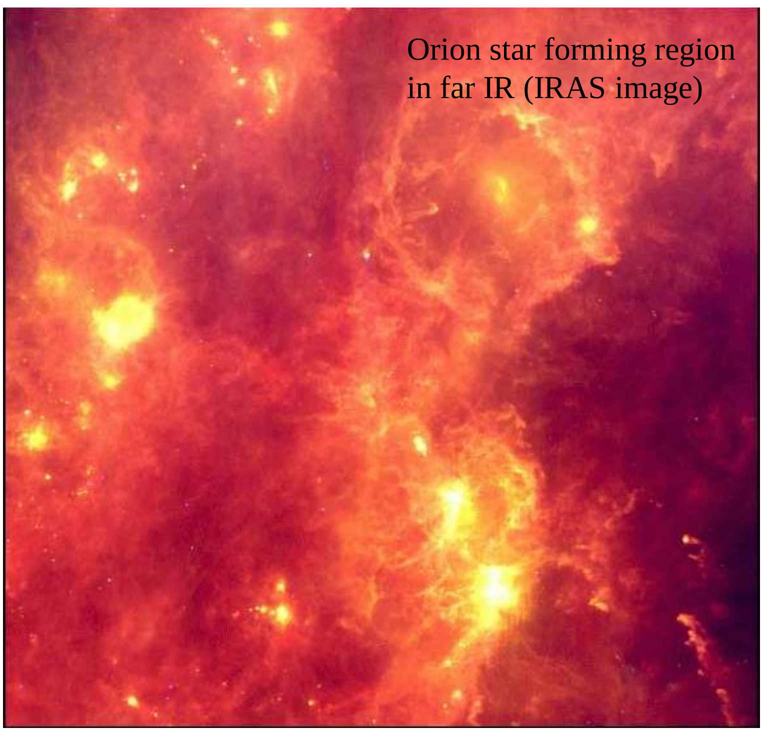
Orion star
forming region

A few pc in diameter
and ~100 solar masses.







Orion star forming region
in far IR (IRAS image)

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M42 Orion Nebula

(Hubble image)

- This is a vast HII region where stars are now being formed although a great deal of the gas has not yet formed into stars.
- Initially stars collapse and radiate because the collapse has heated the gas so much, and so quickly.
- Eventually Nuclear fusion kicks in, the star stabilises and it settles on the main sequence of the HR diagram.

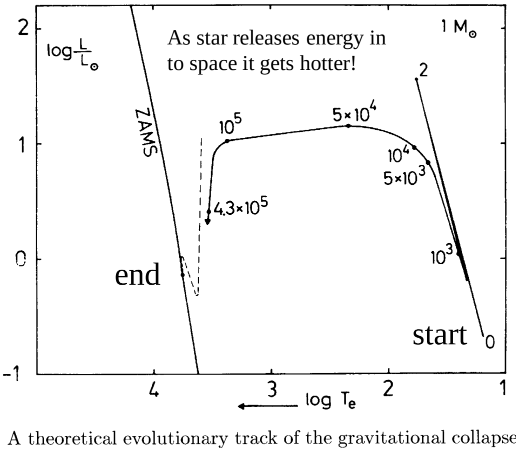


Figure 12.7 A theoretical evolutionary track of the gravitational collapse of a 1 $M_\odot$ cloud through the protostar phase (times are labeled in years since the development of a hydrostatic core). The dashed line shows the quasi-hydrostatic evolution of a pre-main-sequence star, beginning on the Hayashi track. Also shown is a portion of the zero-age main sequence (ZAMS). Both the Hayashi track and the ZAMS are discussed in Section 12.3. (Figure from Appenzeller and Tscharnuter, *Astron. Astrophys.*, *40*, 397, 1975.)

Pre-ZAMS evolution for a cloud of 1 solar mass

Zero age main sequence (ZAMS) is when H first starts to burn in the core of the star. Takes about 0.5 Myr to get there.

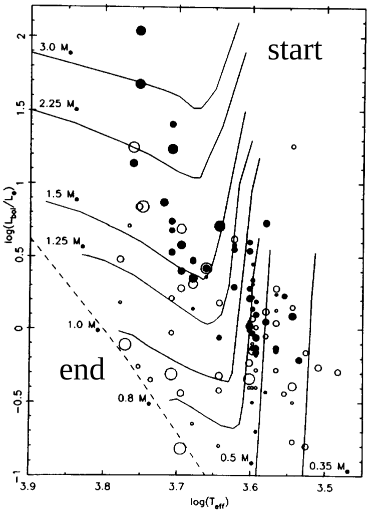


Figure 12.11 The positions of T Tauri stars on the H–R diagram. The sizes of the circles indicate the rate of rotation. Stars with strong emission lines are indicated by filled circles and weak emission line stars are represented by open circles. Theoretical pre-main-sequence evolutionary tracks are also included. (Figure from Bertout, *Annu. Rev. Astron. Astrophys.*, *27*, 351, 1989. Reproduced with permission from the *Annual Review of Astronomy and Astrophysics*, Volume 27, ©1989 by Annual Reviews Inc.)

HR diagram for a star formation region

- T Tauri stars are pre-main sequence stars (stars that have yet to reach the equilibrium that being on the main sequence implies).
- Observational points not very structured because different masses lead to very different collapse luminosities and collapse lifetimes. so hard to compare with theory.

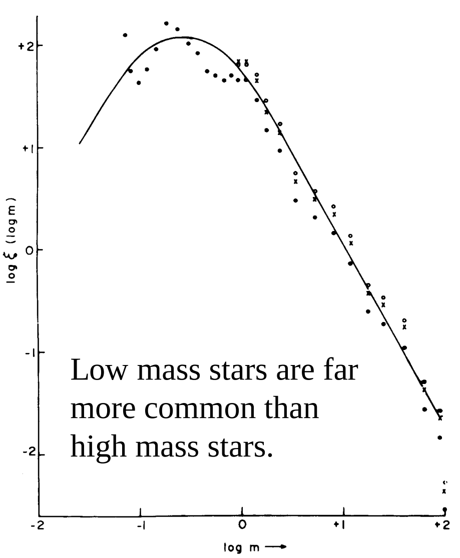
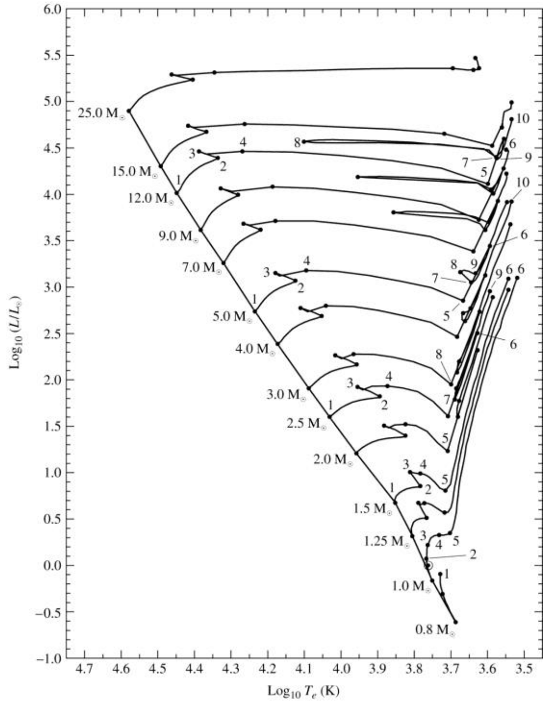


Figure 12.9 The initial mass function, ξ, shows the number of stars per unit area of the Milky Way's disk per unit interval of logarithmic mass that is produced in different mass intervals. Masses are in solar units. (Figure adapted from Rana, *Astron. Astrophys.*, *184*, 104, 1987.)

Initial Mass Function

- We want to know the relative proportions of light and heavy stars.
- Different regions of space, and different star forming regions may give very different results.
- Will depend on the turbulence in the gas clouds out of which the stars are forming.
- This diagram is a composite for the Milky Way.
- Local regions may well be rather different, though the general form will hold since the power spectrum of the turbulence must follow this general form.



<u>How stars evolve post ZAMS</u>

Stars spend ~90% of life on the main sequence.

Mass (M_{sun})	lifetime (M-years)
15	10
3	200
1	9000

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Evolution for low mass stars

When He core gets hot enough for nuclear burning it is still degenerate $\rightarrow$ runaway reaction.

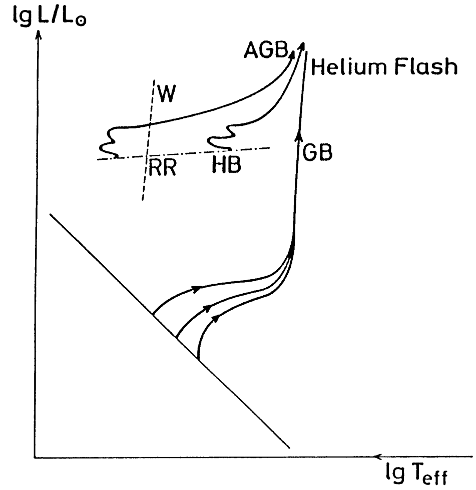


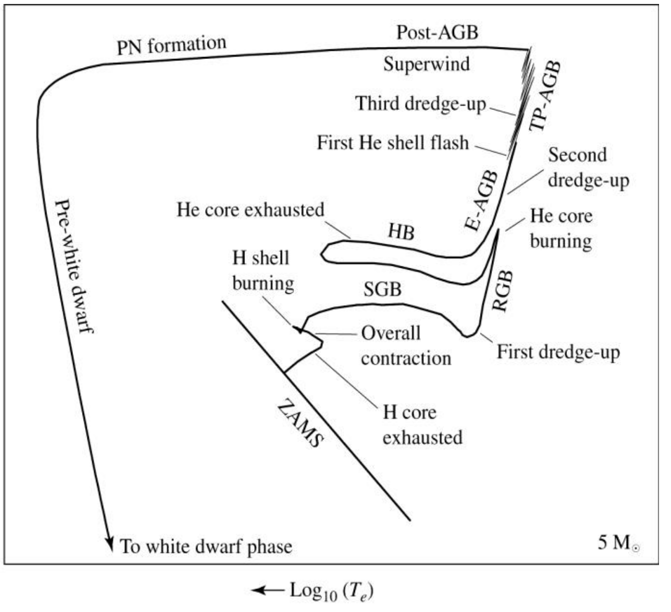
Fig. 32.10. Sketch of the evolution of low-mass stars in the HR diagram. For three slightly different masses the evolutionary tracks in the post-main-sequence merge in the giant branch (GB). After the helium flash they appear on the zero-age horizontal branch (HB), evolve towards the upper right, and merge in the asymptotic giant branch (AGB). The broken line indicates the positions of the variable RR Lyr stars (RR) and of the W Vir stars (W)

$M < 2{\cdot}2\, M_\odot$

Evolution to a planetary nebula and a white dwarf

Large amount of mass loss on the AGB

t=time in years



Evolution to a planetary nebula and a white dwarf

t=time in years

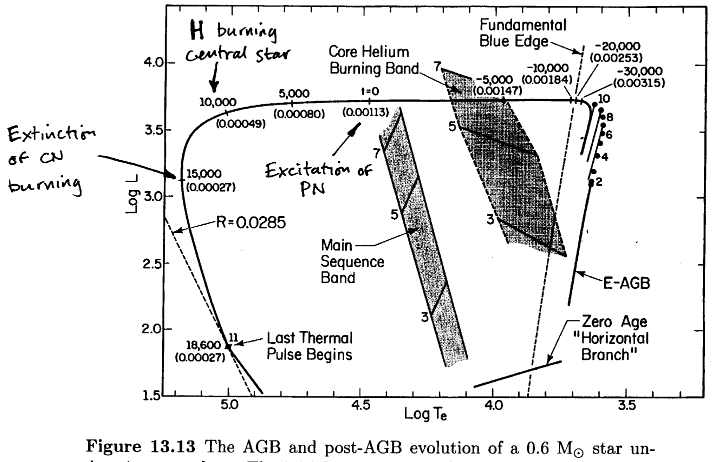


Figure 13.13 The AGB and post-AGB evolution of a 0.6 M☉ star undergoing mass loss. The initial composition of the model is $X = 0.749$, $Y = 0.25$, and $Z = 0.001$. The main-sequence and horizontal branches of 3, 5, and 7 M☉ stars are shown for reference. Details of the figure are discussed in the body of the text. (Figure adapted from Iben, *Ap. J.*, *260*, 821, 1982.)

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Planetary Nebula

The Ring Nebula (HST Image)

Planetary Nebula

The Cat’s Eye Nebula (left) and the Eskimo nebula (right)
Both are HST images.

Planetary Nebula

IC418

(aka The Spirograph Nebula)

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Supernovae

- Once star has an Fe core it cannot burn nuclear fuel at the core and therefore cannot provide enough pressure support.
- Star collapses catastrophically.
- Core eventually is supported by a degenerate neutron gas
- Infalling outer envelope bounces $\rightarrow$ enormous supernova explosion.

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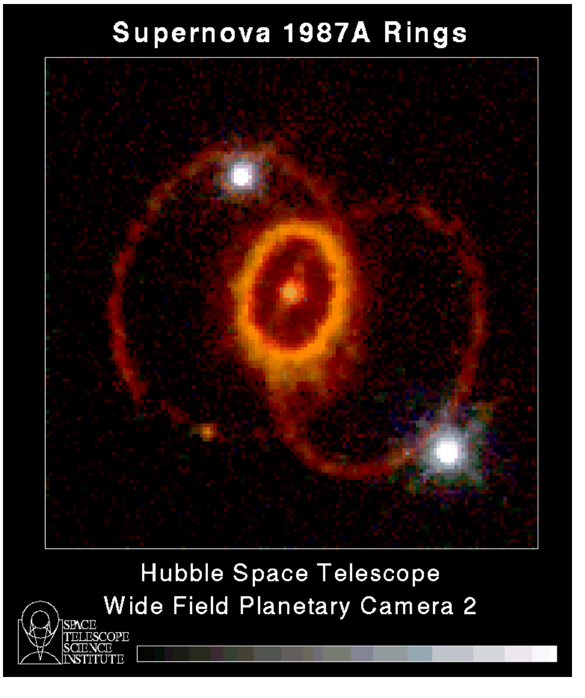
Stellar Evolution Summary

Low mass stars have long lives, high mass ones very short lives.
90% of life spent on main sequence, then red giant then death.

Mass range in solar units	0.5<M<2.2	2.2<M<8	M>8
He flash	Yes	No	No
Mass loss	Small	Medium	Large
Carbon burning	No	No	Yes
Ultimate fate	Planetary Neb +White Dwarf	Planetary Neb +White Dwarf	Supernova Neutron Star or Black Hole

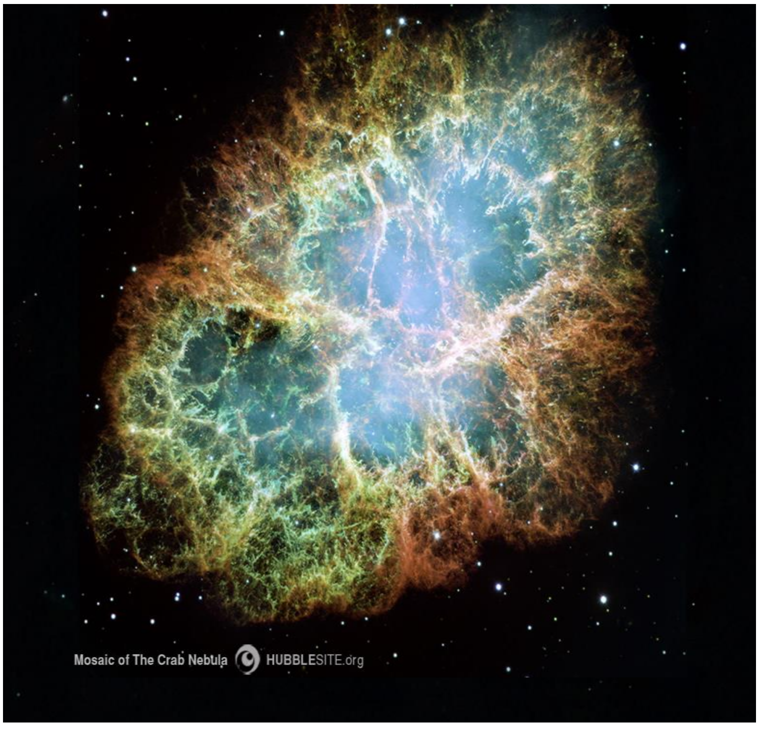
Post Asymptotic Giant Branch (AGB) evolution for massive stars

- For stars with mass > 8-9 M_{sun} we have a supernova.



Crab Nebula, Super-Nova Remnant (SNR)

HST Image.



Crab Nebula, Super-Nova Remnant (SNR)

This is a Lucky Imaging movie of the pulsating star (pulsar) that is the remnant of the supernova of 1054AD

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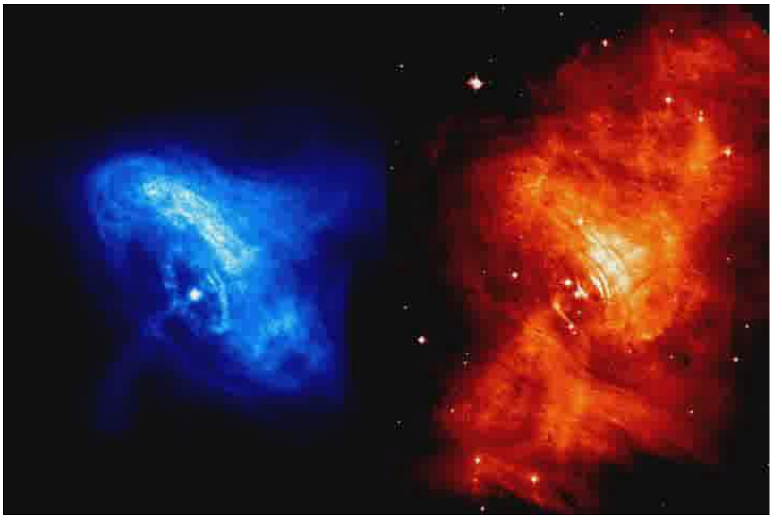
Crab Nebula, Super-Nova Remnant (SNR)

This is an X-ray (Chandra: left) and Optical (HST) movie of the pulsating star (pulsar) and the surrounding nebula that is the remnant of the supernova of 1054AD.



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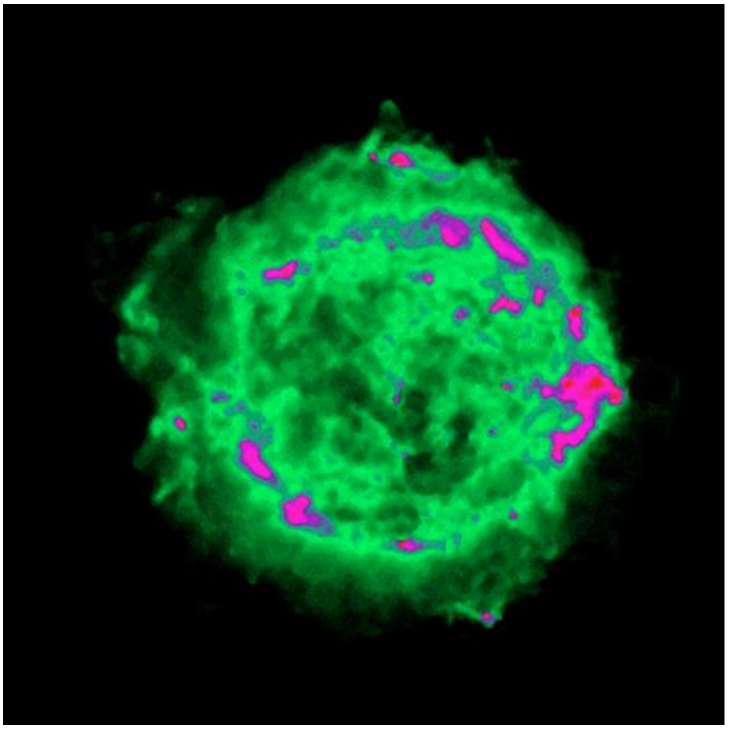
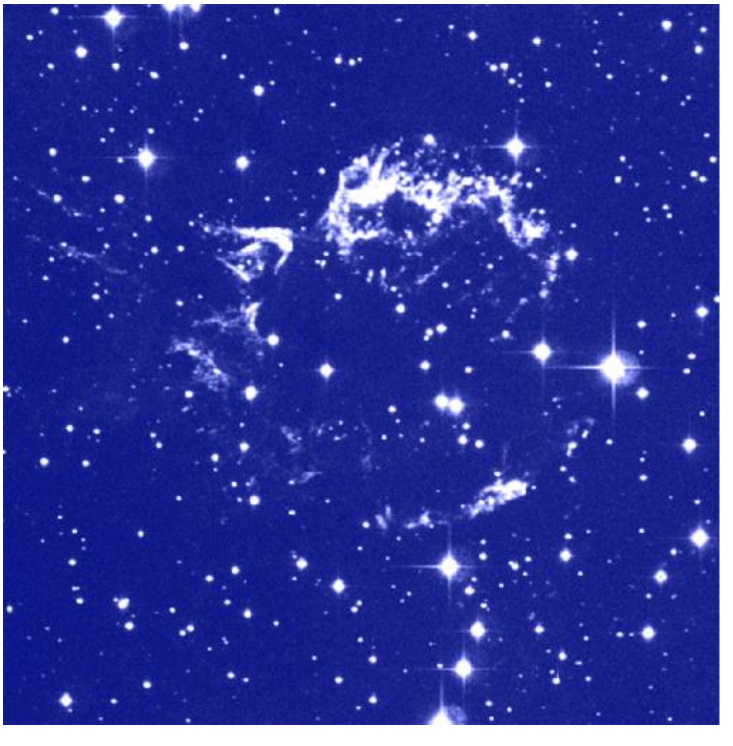
Radio image of Cas-A (SNR)



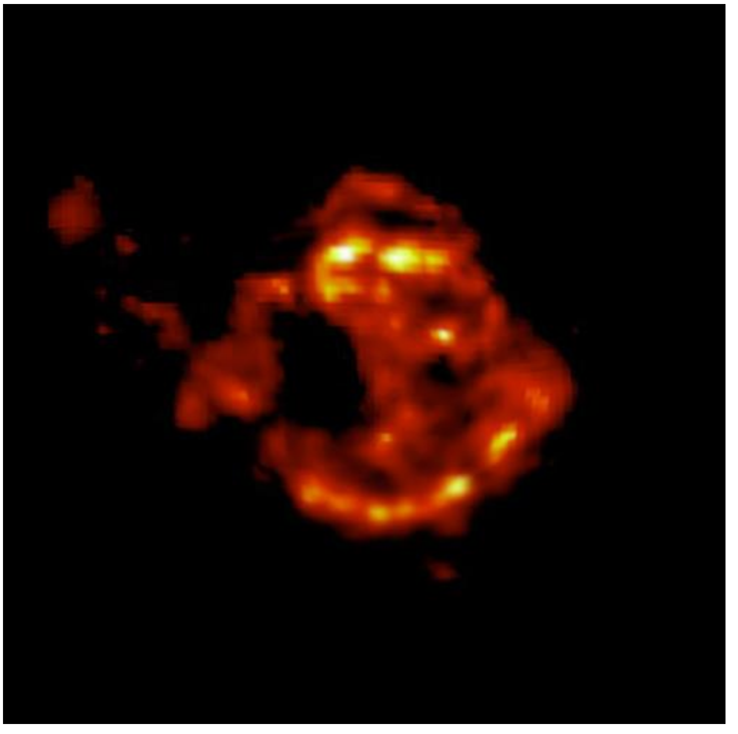
Multi-wavelength X-ray images of Cas-A (SNR)

Top right: composite
Bottom: each atomic species

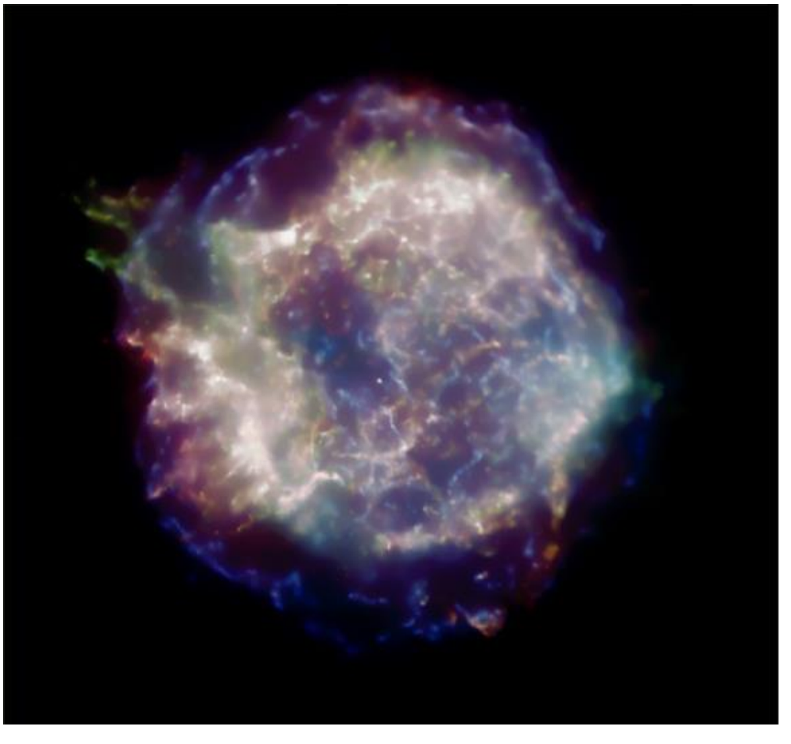
Optical image of Cas-A (SNR)

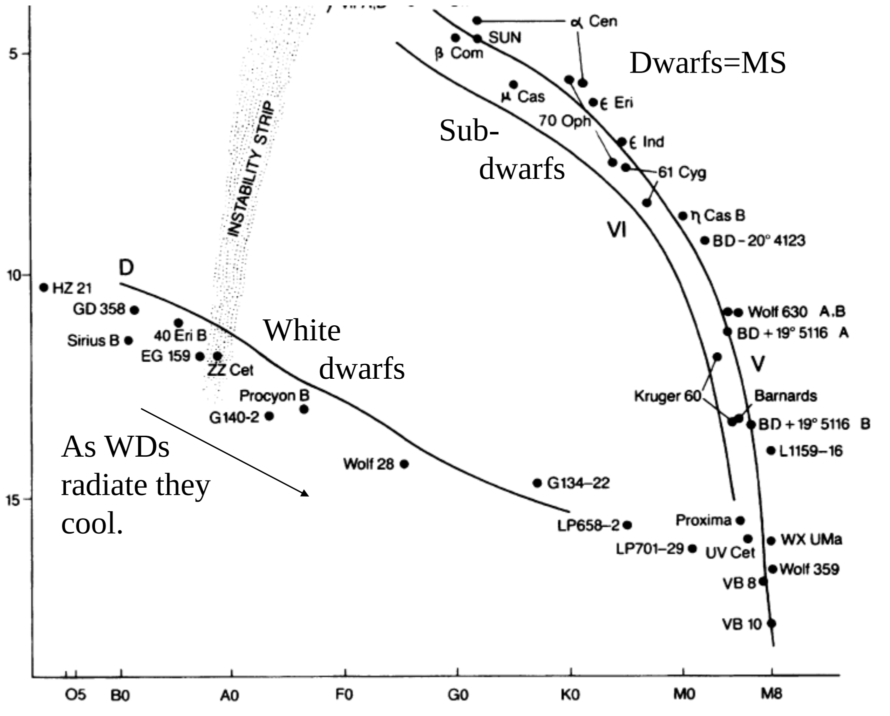


Infra-Red image of Cas-A (SNR)



Chandra (x-ray) continuum image of Cas-A (SNR)





Binary stars

- Most stars are in binary systems.
- Evolution in a binary system can be radically changed by mass exchange, especially when the stars are close together.
- The more massive the star, more likely it will be a binary making them a significant contributor to the energy and momentum input into the interstellar medium.

- We will come back to this later in the course.

Orbits-1: Kepler's Laws

- Kepler's 1st law: A planet orbits the Sun in an ellipse with the Sun at one focus of the ellipse
- Kepler's 2nd law: A line connecting a planet to the Sun sweeps out equal areas in equal time intervals
- Kepler's 3rd law:
  $[\text{Period(years)}]^2 = [\text{semi-major axis(AU)}]^3$ (specific to orbits around the Sun)

Orbits-2: Kepler's 2nd Law

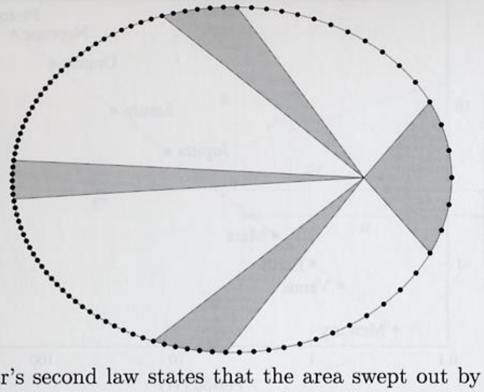


Figure 2.2 Kepler's second law states that the area swept out by a line between a planet and the focus of an ellipse is always the same for a given time interval, regardless of the planet's position in its orbit. The dots are evenly spaced in time.

Orbits-3: Kepler's 3rd Law

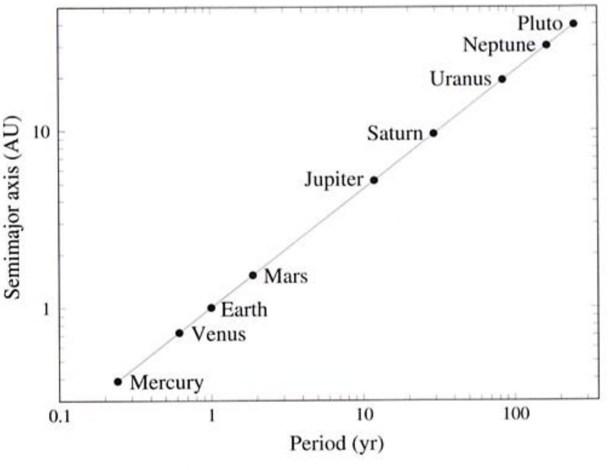


Figure 2.3 Kepler's third law for planets orbiting the Sun.

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Orbits-4: Geometry of an ellipse

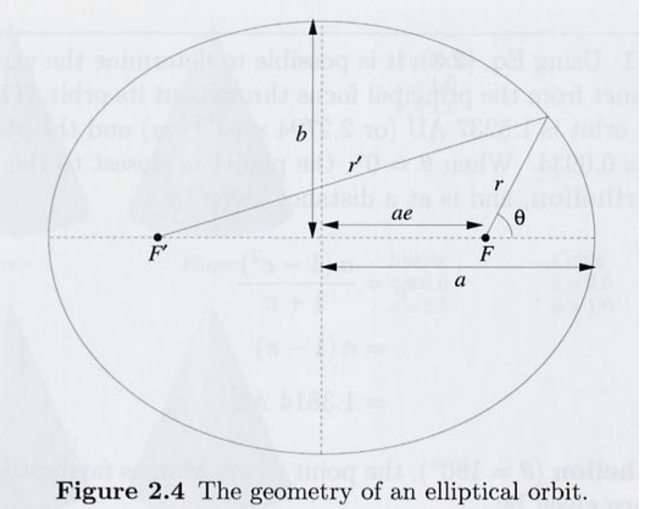

Figure 2.4 The geometry of an elliptical orbit.

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Orbits-5:

Equation of an ellipse : $b^2 = a^2(1\text{-}e^2)$

where a is the semi-major axis and b is the semi-minor axis

equation of an ellipse in polar coordinates

$$(0 \le e < 1) \qquad r = \frac{a(1-e^2)}{1+e\cos\theta}$$

of a parabola $(e=1)$ $\qquad r = \dfrac{2p}{1+\cos\theta}$

where p is the distance of closest approach

of a hyperbola $(e>1)$ $\qquad r = \dfrac{a(e^2-1)}{1+e\cos\theta}$

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Newtonian Mechanics-1

Newton's first law : $\qquad p = mv$

where $p =$ momentum, $m =$ mass and $v =$ velocity

Newton's Second law : $\qquad F = ma$

where $F =$ force and $a =$ acceleration

Universal force of gravity : $\qquad F = G\dfrac{M_1 M_2}{r^2}$

where G is the gravitational constant, M_1 and M_2 are the masses of the 2 bodies and r is the distance between their centres.

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Newtonian Mechanics-2

Centripetal acceleration: $a_c = \frac{v^2}{r}$

where r = radius of circular orbit and v = velocity

Kinetic Energy: $K = \frac{1}{2} m v^2$

Potential Energy: $U = -G \frac{M_1 M_2}{r}$

where G is the gravitational constant, M_1 and M_2 are the masses of the 2 bodies and r is the distance between their centres.

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Newtonian Mechanics-3

Escape velocity: $v_{esc} = \sqrt{2GM/r}$

at a distance r from a body of mass M

Kepler's third law: $P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$

where P = period of orbit and a = semi-major axis of orbit

Virial theorem: Total Energy $= E = \frac{\langle U \rangle}{2} = \langle K \rangle + \langle U \rangle$

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Tides-1

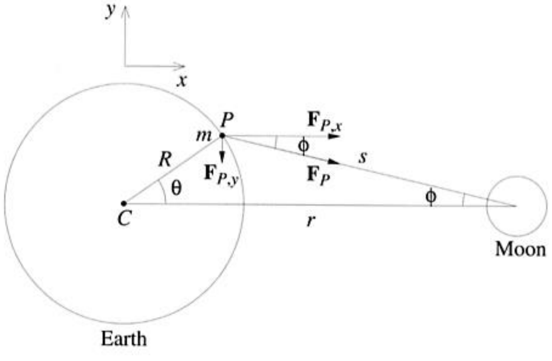

Figure 18.3 The geometry of the tidal force acting on Earth due to the Moon.

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Tides-2

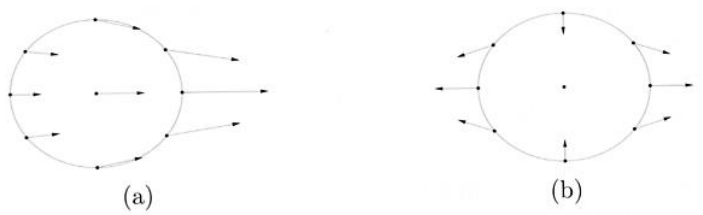

Figure 18.4 (a) The gravitational force of the Moon on Earth. (b) The differential gravitational force on Earth, relative to its center.

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Tides-3

$$\Delta F \approx \frac{GMm}{r^3}\left(2\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

where ΔF is the differential force on a test particle of mass m

$\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions

M is the mass of the Moon

r is the Earth - Moon distance

θ is shown in the diagram

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Tides-4

- Symmetry gives 2 tides every 24h 54m
- Tidal friction causes the Earth to spin down, the day is getting longer by 0.0016 sec/century
- Moon has stopped spinning with respect to Earth
- Moon is drifting away at 3 to 4 cm/year (conservation of angular momentum)
- Spring tide at new/full moon, Sun and Moon work together
- Neap tides at 1st and 3rd quarter

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The Planck Function

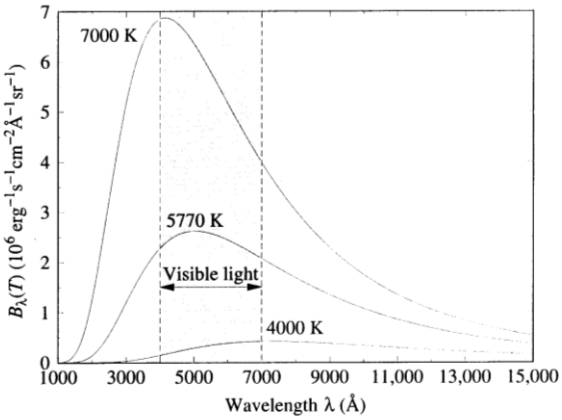
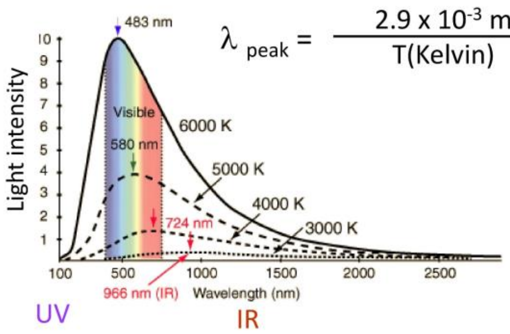


Figure 3.8 Blackbody spectrum [Planck function $B_\lambda(T)$].

Black-body Radiation



$$\lambda_{\text{peak}} = \frac{2.9 \times 10^{-3}\ \text{m}}{\text{T(Kelvin)}}$$

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} * \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1}$$

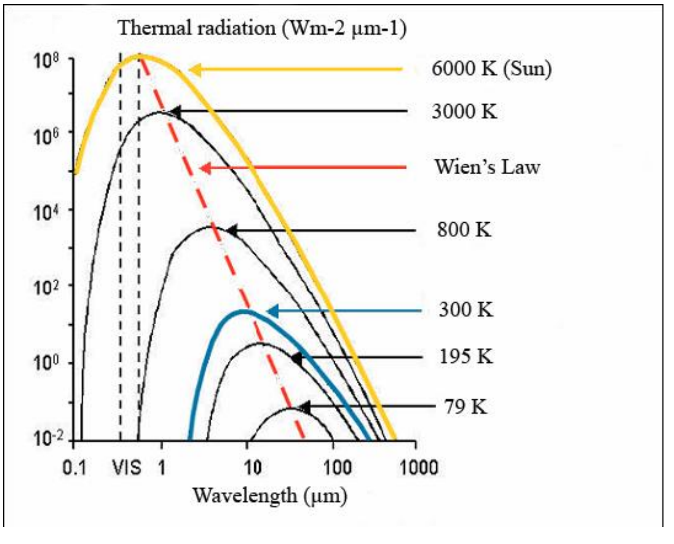
h = Planck's constant = $6.626 * 10^{-34}\ J * s$
c = speed of light = $2.997925 * 10^{8}\ m\ /\ sec$
λ = wavelength (m)
k = Boltzmann's constant = $1.381 * 10^{-23}\ J/K$
T = temperature (K)

Wein's Displacement Law

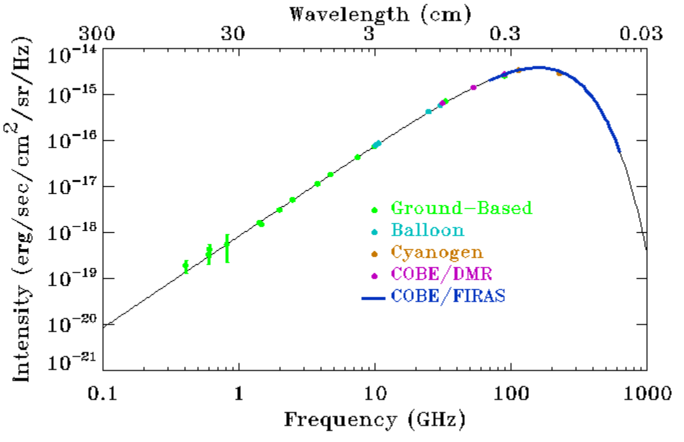
$$\lambda_{\max} T = \text{constant} = 0.290 \ \ (\text{cm K})$$

T(K)	λ_{max}(microns)	Source
20	145	Molecular cloud
293	9.9	Room Temp
1000	2.9	Brown dwarf
3400	0.853	Red giant
5770	0.503	Sun
10000	0.290	O-type star

Wein's Law



Cosmic Background Radiation



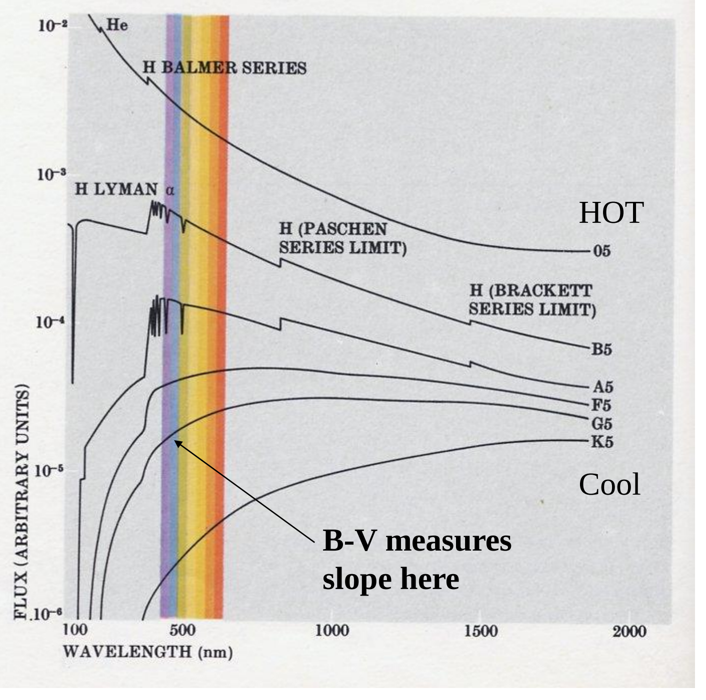
Stephan-Boltzmann Law

The luminosity of a star of radius $= R$ is given by

$$L = 4\pi R^2 \sigma T_e^4 = 4\pi R^2 \int_0^\infty B_\lambda(T)d\lambda$$

where T_e is the effective temperature (by definition)

σ is the Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}



The spectral energy distribution of stars.

They are only approximately blackbodies.

Electromagnetic Radiation

- When a charged particle is accelerated (loses energy) it gives rise to electromagnetic radiation.
- When electro-magnetic radiation is absorbed it accelerates a charged particle.

Absorption cross sections

We can define an absorption coefficient per unit volume $k = n\sigma$

where n is the number of particles per unit volume and σ is the particle cross-section.

If the particles have a velocity v then the interaction rate r is $r = n\sigma v$

The average time between collisions $t = \frac{1}{n\sigma \mathrm{v}}$

The mean free path of a photon is $l = \frac{1}{n\sigma} = \frac{1}{k}$

Opacity (see Stars course) is a mass absorption coefficient with units m^2/kg

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Optical Depth

Optical depth is defined as $$\tau_\upsilon = \int_0^X k_\nu dx$$

or as $$d\tau_\upsilon = k_\nu dx$$

where the radiation of frequency ν has travelled a distance X through a medium with an absorption coefficient k_ν. When τ_ν=1 a factor of 1/e of the radiation is still propagating. For a star we define where τ_ν=1 to be the surface of the star. The apparent size therefore depends on ν.

For τ_ν<<1 we say the material is optically thin

For τ_ν>>1 we say the material is optically thick

The word "optically" applies to any wavelength, from radio to gamma rays.

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Continuum Radiation Types

- ***Blackbody***: emission from an object in ~thermal equilibrium.
- ***Inverse-Compton***: where a photon gains energy after scattering off an energetic electron
- ***Synchrotron***: emission from a particle moving in a magnetic field.
- ***Free-free**:* (Bremsstrahlung – "braking radiation"), where unbound charged particles such as protons or electrons are decelerated.
- ***Bound-free**:* emission caused by ionisation and/or recombination.

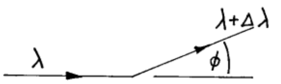
Compton Scattering

This is when a photon interacts with a charged particle and loses energy.

When E=$h\nu \sim m_e c^2$ i.e. a high energy collision then the change in wavelength is

$$\Delta\lambda = \left(\frac{2h}{m_e c^2}\right).\frac{1}{2}\sin^2\varphi$$

where ϕ is the angle through which the photon is deviated.



The Compton wavelength is

$$\lambda_{comp} = \left(\frac{h}{m_e c^2}\right)$$

When E=$h\nu << m_e c^2$ i.e. a low energy collision, we have *Thompson Scattering*.

The Thompson cross section is given by

$$\sigma_T = \frac{8}{3}\pi r_0{}^2 = \frac{8}{3}\pi\left(\frac{e^2}{m_e c^2}\right)^2$$

$$= 6.65\times10^{-29}\ \text{m}^2$$

This is important for high temperature regions inside stars

Inverse-Compton Radiation

If the photon gains energy instead of losing it we create inverse-Compton radiation.

$$\frac{\nu_{after}}{\nu_{before}} = \gamma^2$$

Where γ is the Lorentz factor of the energetic electrons that are interacting:

$$\gamma = 1 \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

When $\gamma > 10^3$ radio waves can be shifted in frequency by more than a factor of one million. Also, optical and UV radiation can become X-ray radiation.

This is an important process in Active Galactic Nuclei (AGN).

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Bremsstrahlung

- The dominant luminous component in a cluster of galaxies is the 10^7 to 10^8 Kelvin intracluster medium. The emission from the intracluster medium is characterized by thermal *Bremsstrahlung*.
- Thermal *Bremsstrahlung* radiation occurs when the particles populating the emitting plasma are at a uniform temperature and are distributed according to the Maxwell–Boltzmann distribution

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left[\frac{-mv^2}{2kT}\right]$$

The bulk emission from this gas is thermal *Bremsstrahlung*.

$$\varepsilon_{ff} = 1.4 \times 10^{-27} T^{1/2} n_e n_i Z^2 g_B$$

where 'ff' stands for free-free, n_e and n_i are the electron and ion densities, respectively.

Z is the number of protons of the bending charge, g_B is the frequency averaged Gaunt factor and is of order unity, and T is the global x-ray temperature determined from the spectral cut-off frequency

$$\hbar\nu = kT$$

above which exponentially small amount of photons are created because the energy required for creation of such a photon is available only for electrons in the tail of the Maxwell distribution.

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Synchrotron Radiation-1

- The force on a charge q is $F = qE + \frac{\mathrm{v}}{c} B$
- Where E is the electric field vector, B is the magnetic field vector and v is the velocity vector.
- We are mostly concerned with synchrotron radiation due to electrons because they have a large charge to mass ratio, but we could also have synchrotron radiation from protons.
- When the electron moves parallel to the magnetic field lines it experiences no force.
- When the electron moves across the magnetic field lines it experiences a force which causes it to move in a circle.
- As the electron spirals it loses energy. Moves in smaller and smaller circles. Combined with a component of motion along the field lines it moves along a helix with a decreasing radius. Orbital revolution has a cyclical frequency $\omega_c = 2\pi \mathrm{v}_c$

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Synchrotron Radiation-2

- and an orbital radius of $r_c = \frac{\mathrm{v}}{\omega_c}$
- for relativistic electrons $m = \frac{m_e}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} = m_e \gamma, \quad E = mc^2 = m_e \gamma c^2$
- If we equate the centrifugal force to the force from the magnetic field we get:

$$
\begin{aligned}
& e\mathrm{v}B = m\mathrm{v}^2 / r_c \\
\Rightarrow \quad & eB = m\mathrm{v} / r_c \\
& \omega_c = \frac{\mathrm{v}}{r_c} = \frac{eB}{m} = \frac{eB}{m_e \gamma}
\end{aligned}
$$

- In the rest frame of the electron the emitted radiation is a simple dipole.
- However, for a relativistic electron in the observers rest frame the radiation is "beamed" into a cone in the direction in which the electron is travelling.

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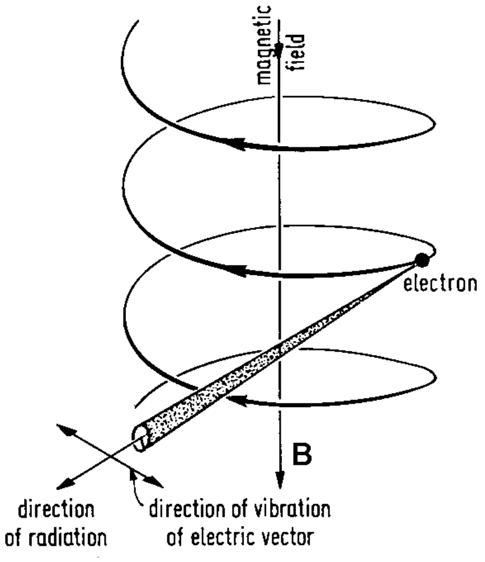
Synchrotron Radiation-3



Beaming of synchrotron radiation

Beam half angle

$$\theta = \frac{m_e c^2}{E} \sim \frac{1}{\gamma}$$

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Synchrotron Radiation-4

- The observer therefore only sees radiation for a fraction of the orb
- It's a bit like a lighthouse. An electron will radiate at a characteristic frequency

$$\nu \sim 300\gamma^2 B$$

- To get the spectrum of a whole ensemble of electrons spiralling in a magnetic field we integrate over all the pitch angles and all the electron energies.
- Extragalactic radio sources show spectra of the form

$$\text{Flux} \quad = F(\upsilon) \propto \upsilon^{-\alpha}$$

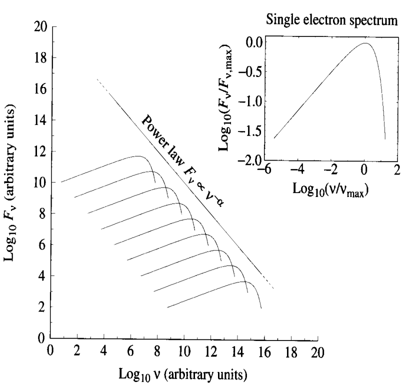

Figure 26.5 The power-law spectrum of synchrotron radiation, shown as the sum of the radiation produced by individual electrons as they spiral around magnetic field lines. The spectrum of a single electron is at the upper right. The turnover at low frequencies due to synchrotron self-absorption is not shown.

- This is a power law with a ***spectral index*** α.

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Synchrotron Radiation-5

- A flux distribution with a power-law form is consistent with a simple power-law distribution of electron energies. $N(E)dE \propto E^{-\beta}dE$

  with $\alpha = \dfrac{\beta\text{-}1}{2}$

- Typically the spectral index for radio sources is between 0.5 and 2.
- Synchrotron radiation can also be seen in the optical and the UV. The rate at which electrons loose energy goes as γ^2 and something has to drive the process to keep it going.
- Without a driving energy source the max frequency or cut-off frequency reduces with time

$$\nu_{cut} \sim 10^{12} \frac{1}{B^3 t^3} \quad \text{Hz}$$

- Below a frequency ν_{SA} the region is optically thick because electrons interact with low energy photons (they pick up energy from them).
- This is called *synchrotron self-absorption.* Below ν_{SA} we have

$$\text{Flux} \quad = F(\upsilon) \propto \upsilon^{+2.5}$$

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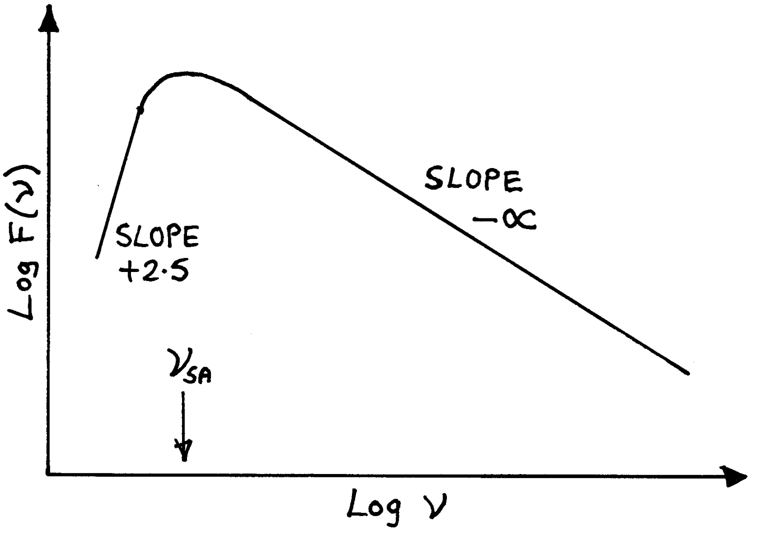
Synchrotron self-absorption



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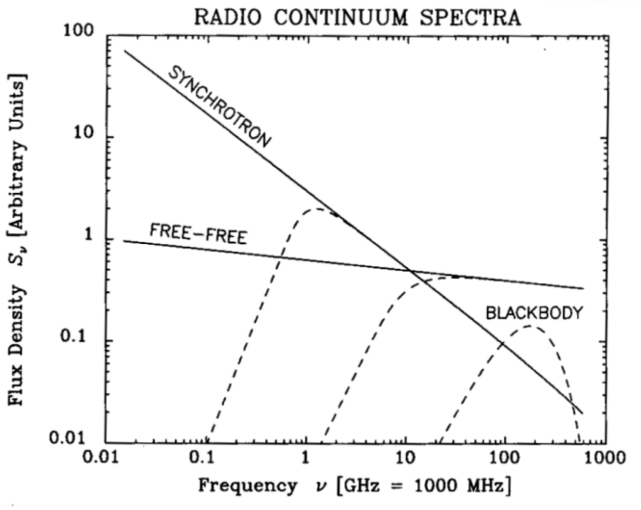

Figure 1. Indicative spectra for the principal continuum emission mechanisms in cosmic radio sources. Solid curves indicate spectra at frequencies for which the emitting component is transparent (negligible self-absorption); dashed curves, opaque (substantial self-absorption). Spectra of actual cosmic sources are often more complex, owing to multiple components or other inhomogeneous structure.

Types of radio continuum

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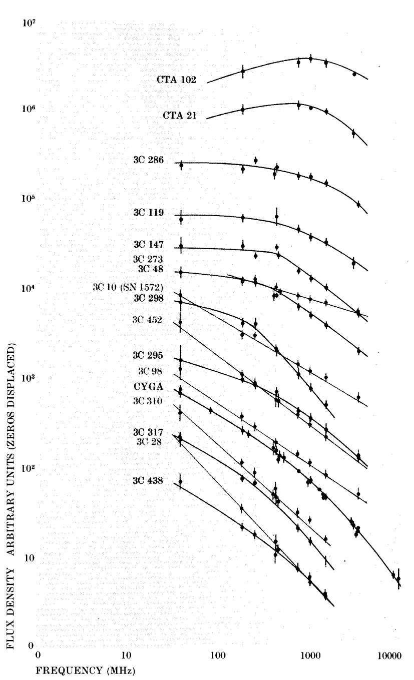

Example spectra of radio sources

18.11: *Spectra of several radio sources on a logarithmic plot of intensity against frequency. The characteristic shape of these spectra is due to emission by electrons in a synchrotron process.*

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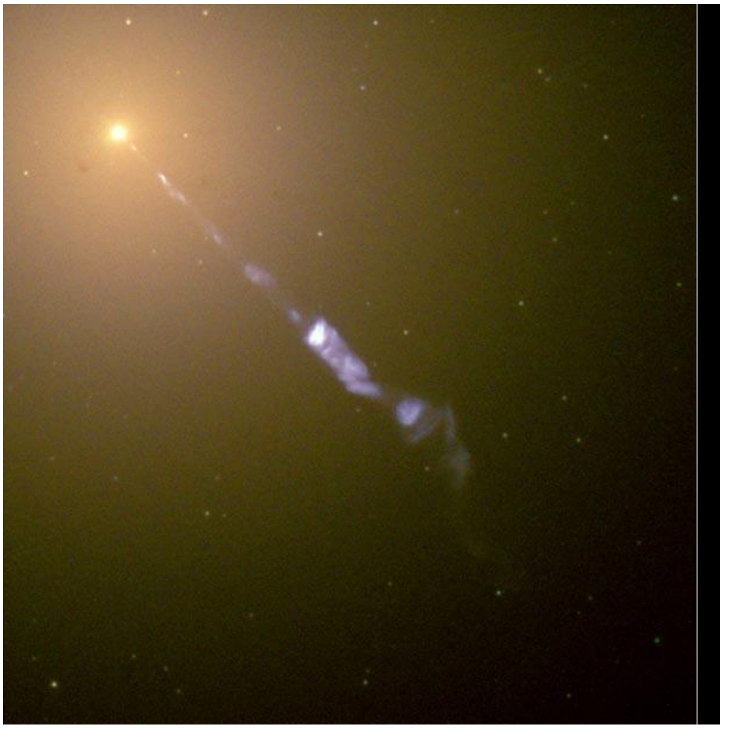

M87 –JET

This optical image (from HST) shows the jet which is emitting synchrotron radiation

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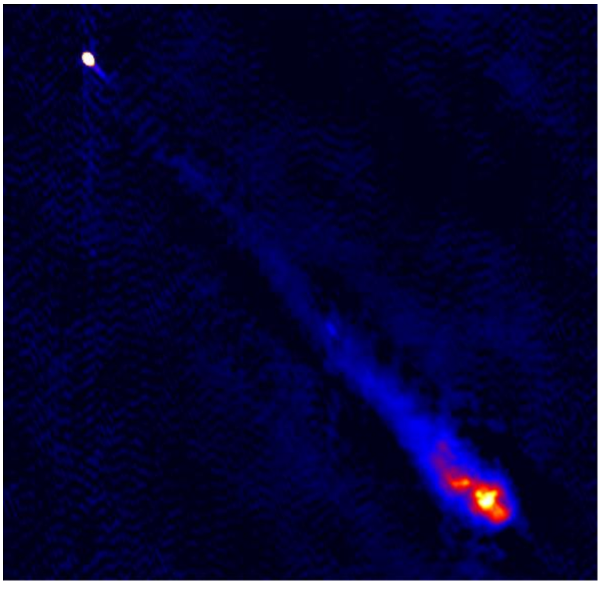
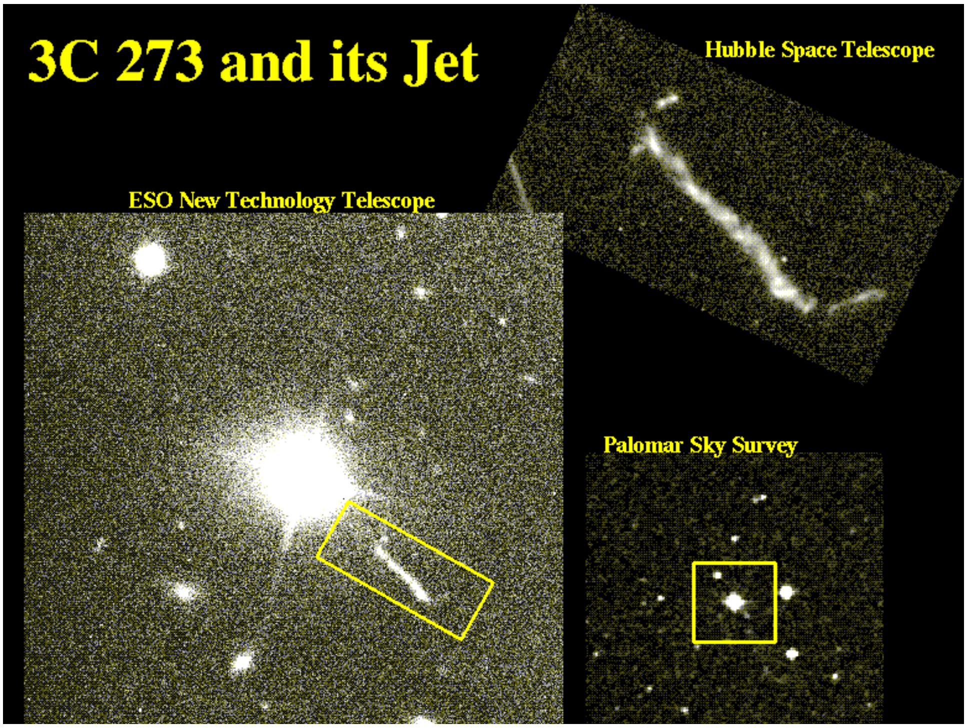
Quasar 3C273 and its jet

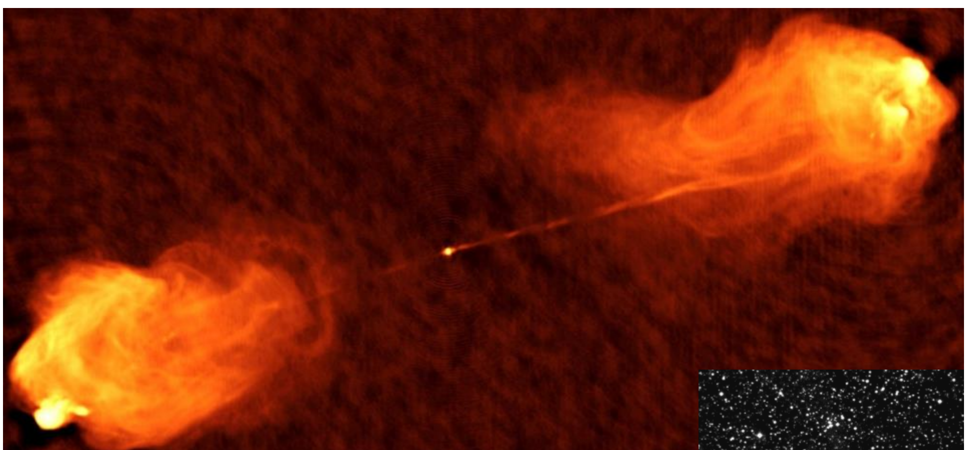
Radio image taken with MERLIN

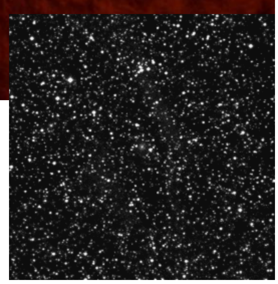


Optical image



Cygnus-A, jets and radio lobes





One of the brightest radio sources in the sky (above in a VLA radio image), compared with the totally insignificant optical image (right) shows how remarkable this object actually is. At a distance of 230 Mpc it is 120 kpc in overall size.

Line spectra

- Atoms and molecules have quantized energy states.
- A change between two states is associated with radiation of a specific wavelength.
- An emission line results when the atom or molecule looses energy.
- An absorption line results when the atom or molecule gains energy.
- For atoms the energy levels are electronic.
- For molecules the energy levels can be due to rotation and vibration as well.
- Atomic transitions have higher energies generally than molecular ones.

Atomic transitions produce UV, optical, IR emission.
Molecular transitions produce IR, mm, radio emission.

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Transition Probabilities

- Transitions between levels follow quantum mechanical selection rules.
- Using these rules one can calculate the probability that an atom in isolation will undergo a particular (downward) transition.
- For an atom which is perturbed by a neighbour, energy can be exchanged without the absorption or emission of a photon (collisional excitation/de-excitation).
- Have to consider in any particular environment whether a collision or a transition is more likely.

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Atomic hydrogen spectrum

- Lyman series comes from transitions involving the ground state (n=1).
  Lyman-α is between n=1 and n=2,
  Lyman-β is between n=1 and n=3,
  Lyman-γ is between n=1 and n=4, etc.

- Balmer series comes from transitions involving n=2 and higher states.

- Paschen series comes from transitions involving n=3 and higher states.

- Brackett series comes from transitions involving n=4 and higher states.

- The *limits* of a series correspond to the transition from that level to $n=\infty$.

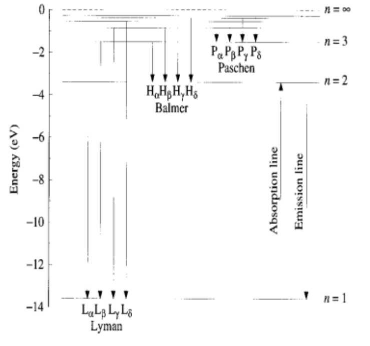

Figure 5.7 Energy level diagram for the Balmer lines (downward arrows for emission lines, upward arrow for absorption lines).

Ly-α 1215 Å	Ly-limit ~920Å
H-α 6563 Å	Balmer-lim 3646Å
Pa-α 18751Å	Pa-limit ~9500Å

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Terminology

- HI=H^0=neutral hydrogen

- HII=H^+=ionised hydrogen

- H_2 is molecular hydrogen

- CIV is an ionised carbon atom with 3 electrons removed (=C^{+3})

- [OII] refers to a forbidden line from singly ionised oxygen, e.g. the line [OII]3727 which has a wavelength of 3727Å (372.7nm).

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UV spectra of hot stars

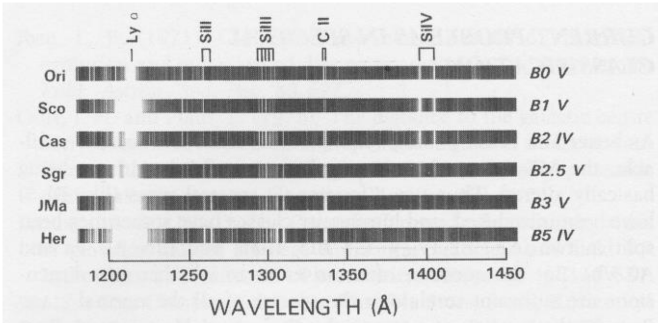

Figure 2. A sequence of B-star spectra taken by the *International Ultraviolet Explorer* satellite in the high-dispersion mode. The spectra have been resampled to a resolution of 0.25 Å and artificially widened by pixel replication. The most prominent spectral features in the 1200–1450 Å region are marked, but these are not necessarily the criteria used for spectral classification. (*From Criteria for the Spectral Classification of B Stars in the Ultraviolet, by J. Rountree and G. Sonneborn,* Astrophysical Journal, *March 1991. Reprinted by permission of the authors.*)

Highly ionised species.

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Zeeman Splitting

- The energy levels associated with different angular momentum quantum numbers are normally the same.
- In the presence of a weak magnetic field they are different giving rise to the *normal Zeeman effect*.
- Other quantum numbers can give rise to different energy levels in the presence of strong magnetic fields giving the *strong Zeeman effect*.
- Useful probe of magnetic fields, e.g. sunspots.

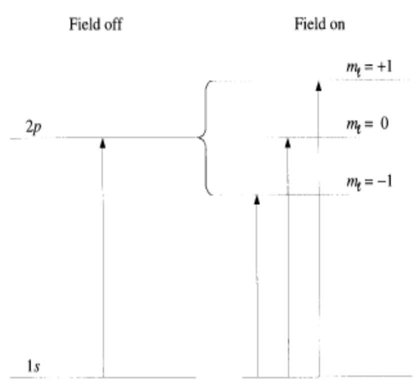

Figure 5.13 Splitting of absorption lines by the Zeeman effect.

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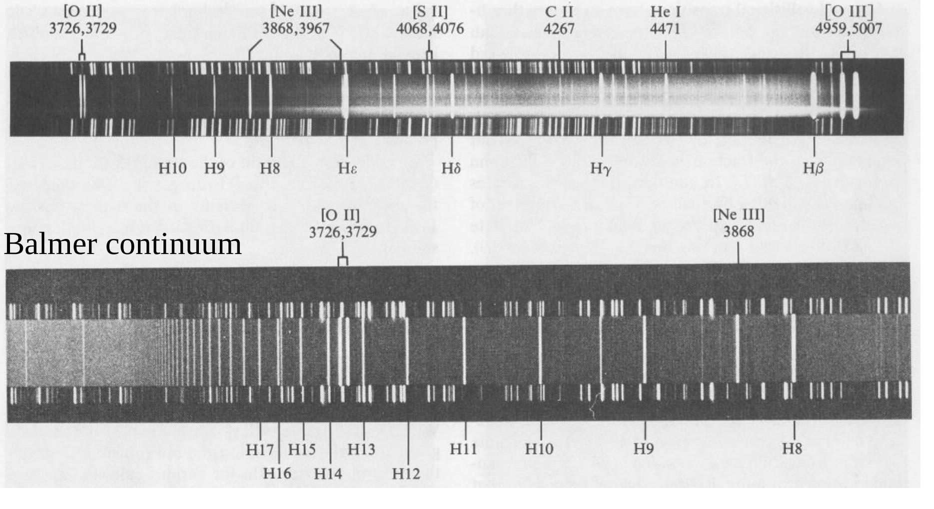
Forbidden lines

- In astronomy lines are seen which cannot be created in the laboratory because their transition probabilities are too low. These are called "forbidden lines".
- Despite the very long lifetime of an electronic state the large volumes and low densities in space yield a large number of transitions to a lower energy state at any one time.
- In the lab collisional de-excitation prevents the lines being seen.

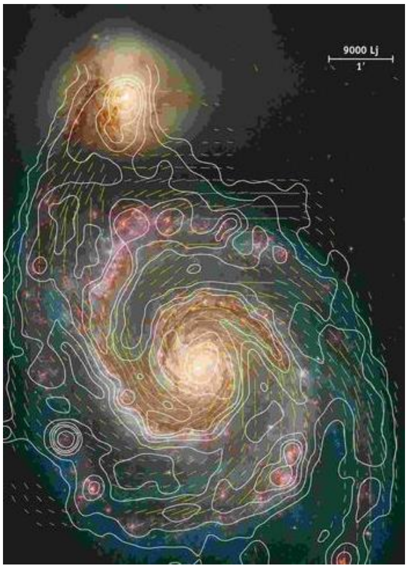
- Example here is the Orion nebula Where most of the radiation is in forbidden lines

Orion Nebula Spectrum (HII Region)



HI – 21cm radio wavelength line

- Neutral hydrogen in the ground state has two possible spin quantum numbers.
- When the spin of the electron is aligned with the spin of the proton the energy is slightly higher than when they are anti-aligned.
- When the electron flips from high to low states (lifetime ~ million years) we get a very low energy photon which can be observed at 21-cm (1420 MHz) with radio telescopes.
- Collisional de-excitation is more likely (every hundred years) but enough survive to make this a very important tracer of neutral hydrogen.
- This is a radio map at 21 cm of the spiral galaxy M51 which has a companion.



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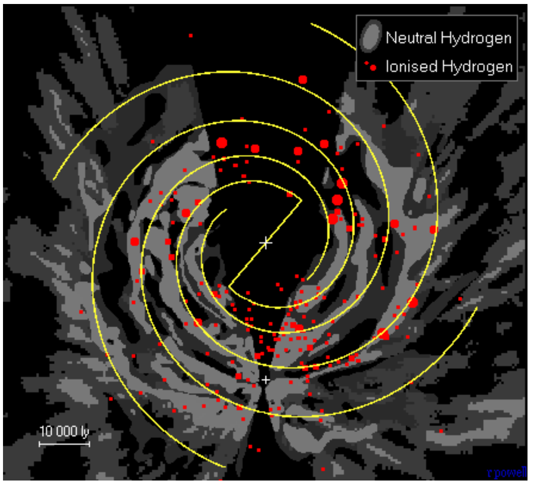

The Milky Way

We can also trace the neutral hydrogen in our own galaxy.

This is a 21-cm map of the spiral structure of our galaxy.

Our sun is marked by a cross. It lies ~8000 pc from the centre of the galaxy.

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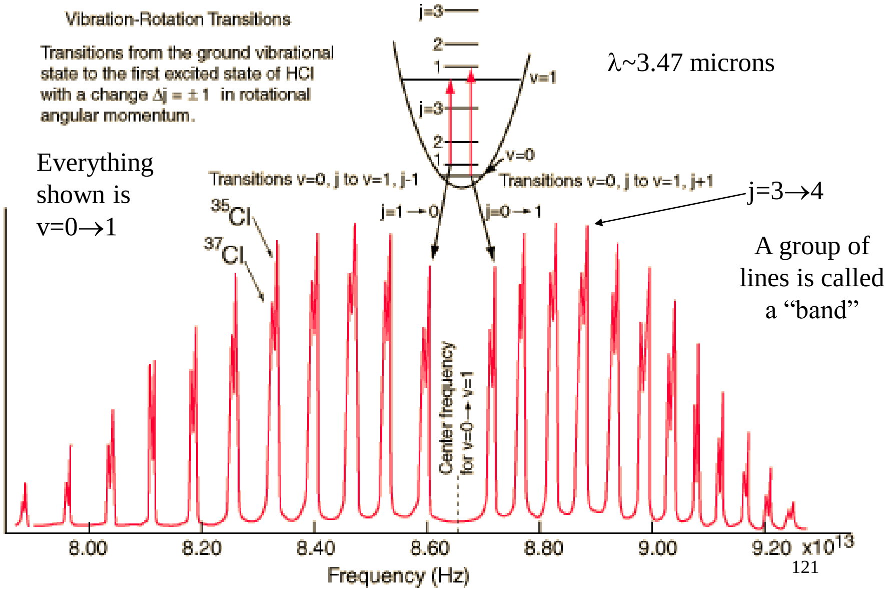
Molecular spectrum – HCl in lab



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Continuum radiation - examples

Type of radiation	Examples
Black - body	Anything optically thick, stars, dense clouds
Inverse Compton Scattering	X-rays and Gamma rays from Active Galactic Nuclei (AGNs)
Synchrotron	Radio sources, jets, radio lobes, SNRs
Free - free	X-ray clusters, radio emission from HII regions, solar flares
Bound - free	HII regions

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Stellar spectral types

- Letters denote various classes which essentially measure temperature.
- Weird sequence is a throwback to the early days of the field when they didn't understand the underlying physics.
- Basic sequence goes O, B, A, F, G, K, M with O being the hottest and M being the coolest.
- Each class is divided into 10 subclasses, e.g. the Sun is a type G2
- Brown dwarfs, Carbon stars and white dwarfs classified separately.

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Boltzmann's equation

- This equation lets us calculate the number of atoms in various states of *excitation.*

$$\frac{N_a}{N_b} = \frac{g_b}{g_a} \exp\left[-(E_b - E_a)/kT\right]$$

where N_a and N_b are the numbers of atoms in the electronically excited states *a* and *b.*

- E_a and E_b are the energies of the electronically excited states *a* and *b.*
- Each energy level can have several quantum states and to allow for this we use the statistical weights g_a and g_b.

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Saha's equation

- This equation lets us calculate the number of atoms in various states of *ionisation*.

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(\chi_i / kT\right)$$

where N_i is the number of atoms in ionisation state *i*.

- Z_i is the partition function for ionisation state i.
- The partition function is to take in to account the number of different electronic excitation states an ion can have.
- χ_i is the energy needed to ionise the atom, $i \rightarrow i+1$
- m_e is the mass of an electron.

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Basis of stellar spectral classes

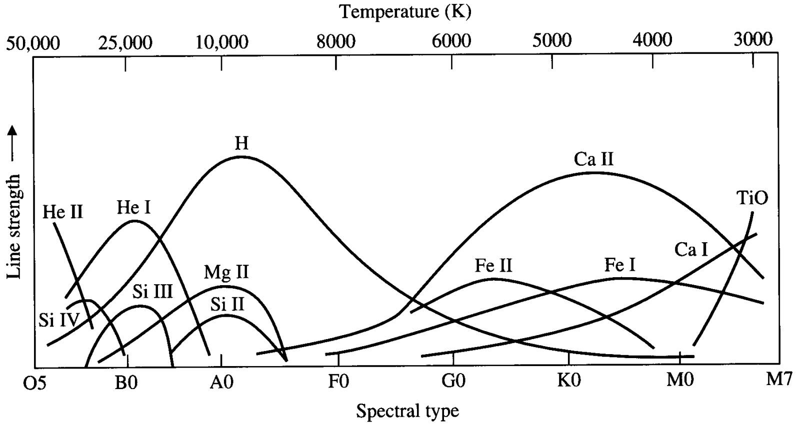

Figure 8.9 The dependence of spectral line strengths on temperature.

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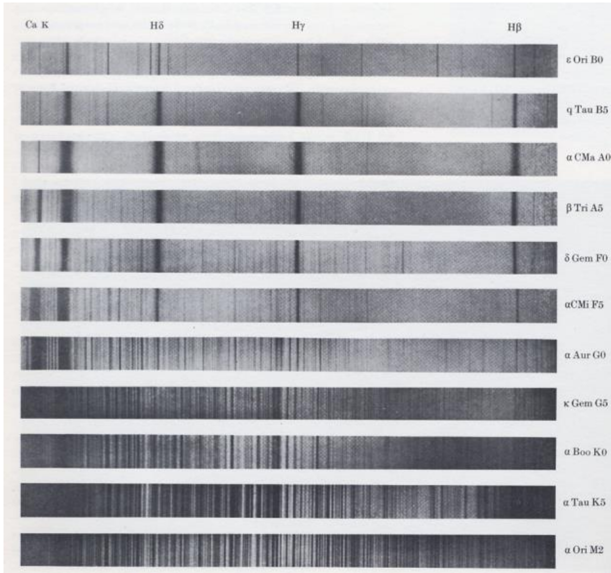
Spectral classification main sequence stars

These spectra only run in the blue, from 390nm to 490nm



Hot

Cool

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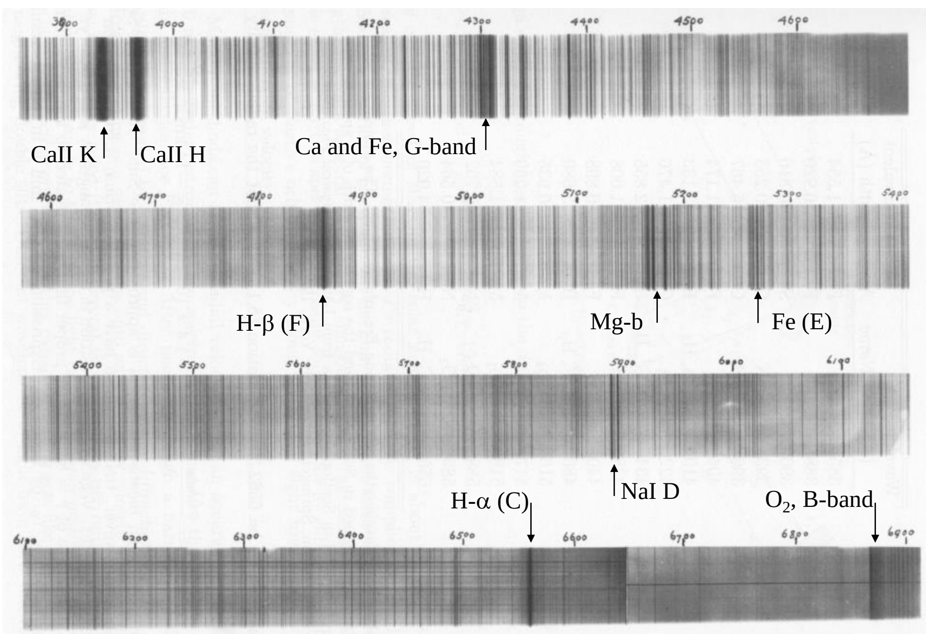
The spectrum of the Sun (385 to 690 nm) (3850-6900 Angstroms)



The Doppler Effect

For non-relativistic velocities

$$\frac{\lambda_{obs}-\lambda_{lab}}{\lambda_{lab}}=\frac{\Delta\lambda}{\lambda_{lab}}=\frac{v_r}{c}$$

where λ_{lab} is the wavelength of the line from lab measurements (i.e. at rest), λ_{obs} is the wavelength of the line as observed and v_r is the velocity along the line of sight (r stands for radial).

When the line is shifted to longer wavelengths (red-shifted) we have a positive radial velocity. A blue-shift corresponds to a negative radial velocity.

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Sodium (Na) D lines, Interstellar Absorption

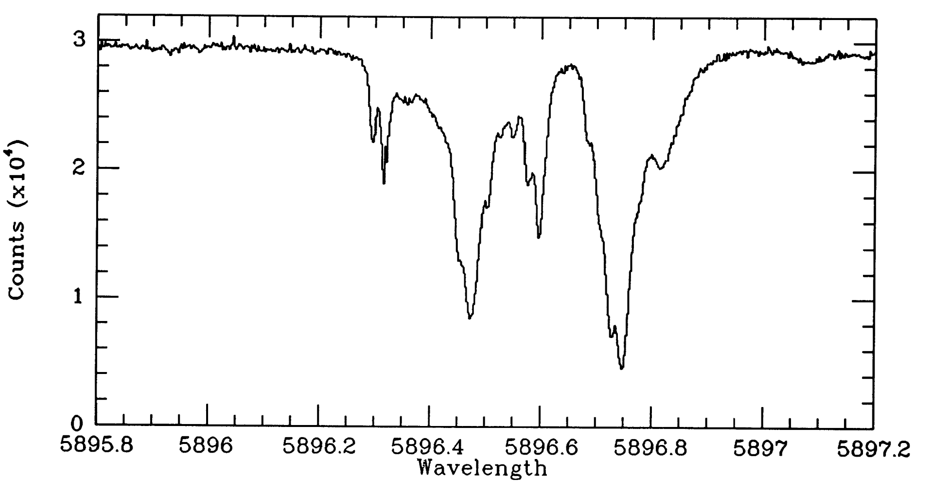

Figure 13. Spectrum of ϵ Orionis obtained also in January 1994. Na D1 shows a complex structure from several clouds with hyperfine splitting evident in most of them. Compare this spectrum with figure 8.

The Lyman-alpha forest in a QSO spectrum

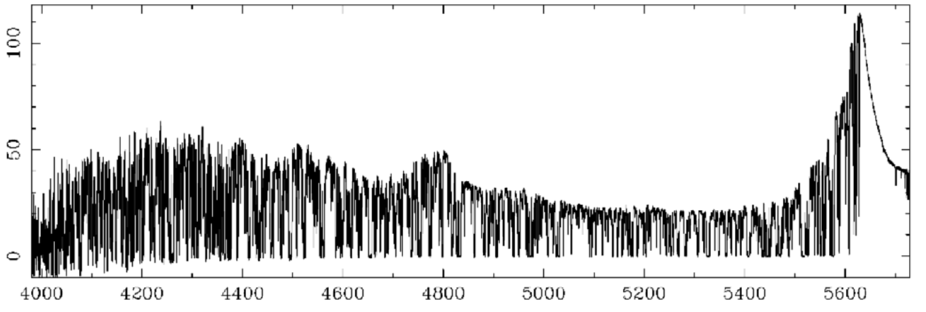

Figure 1 High resolution [full width at half maximum (FWHM) $\approx$ 6.6 km s^{-1}] spectrum of the z_{em} = 3.62 QSO1422+23 (V = 16.5), taken with the Keck High Resolution Spectrograph (HIRES) (signal-to-noise ratio ~150 per resolution element, exposure time 25,000 s). Data from Womble et al (1996).

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Damped Ly-α systems in QSO spectra. Clouds of neutral H in the inter-galactic medium

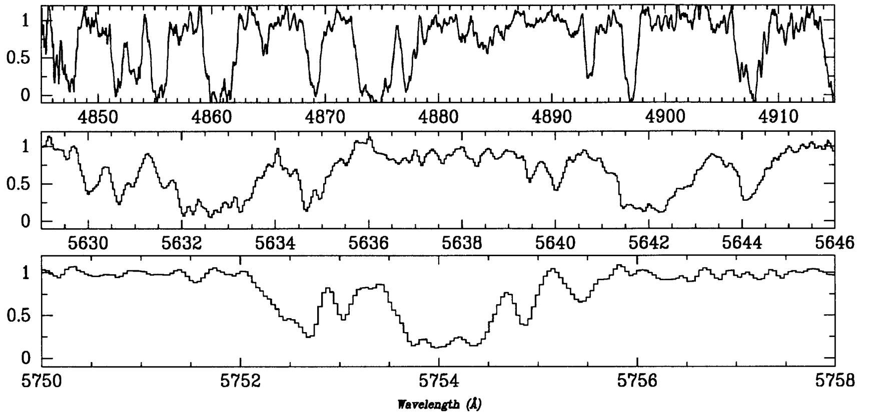

Figure 7: *Top*: A 70 Å portion of the Lyman-alpha forest at $z \approx 3$ in PKS 2126-158. *Center*: C IV λλ1548,1550 doublet at $z = 2.638$ in PKS 2126-158. *Bottom*: Si II λ1526 at $z = 2.769$ PKS 2126-158.

Line widths and profiles

- *Natural broadening:* ΔE for an electronic orbit due to uncertainty principle. Gives a line width of ~5×10^{-4} Å.
- *Doppler broadening:* In a stellar atmosphere from Maxwellian velocity distribution (0.5Å for the sun). Stellar rotation and turbulence can also contribute. In a stellar system due to velocity distribution of the stars (~5Å in a galaxy at 5000Å implies ~300 km/sec).
- *Pressure broadening:* In stellar atmospheres, for example. Electric field of nearby atoms perturbs the orbital energy levels. Has a big effect on the wings of line profiles.

- As abundance of a species grows absorption lines become stronger but will eventually saturate as in these Voigt profiles.

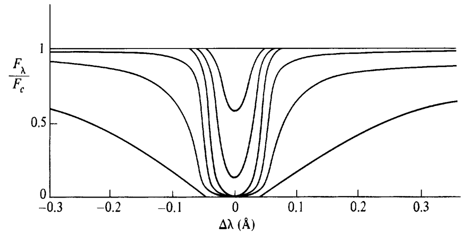

Figure 9.20 Voigt profiles of the K line of Ca II. The shallowest line is produced by $N_a = 3.4 \times 10^{11}$ ions cm^{-2}, and the ions are ten times more abundant for each successively broader line. (Adapted from Novotny, *Introduction to Stellar Atmospheres and Interiors*, Oxford University Press, New York, 1973.)

Luminosity classes

- Ia, Ib, super-giants, e.g O5Ia
- II bright giants
- III normal giants
- IV sub-giants
- V dwarfs (main seq. stars), e.g. G2V the sun
- VI sub-dwarfs
- WDs white dwarfs

- Top spectrum is the biggest star, bottom is the smallest.

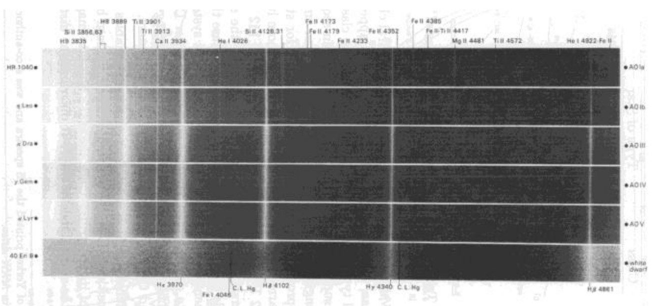

Figure 8.14 A comparison of the strengths of the hydrogen Balmer lines in types A0 Ia, A0 Ib, A0 III, A0 IV, A0 V, and a white dwarf, showing the narrower lines found in supergiants. (Figure from Yamashita, Nariai, and Norimoto, *An Atlas of Representative Stellar Spectra*, University of Tokyo Press, Tokyo, 1978.)

Equivalent widths

- The equivalent width is defined as the width a line of the same area would have if it had a square line profile.

$$W = \int \frac{F_c - F_\lambda}{F_c} d\lambda$$

- where F_c is the continuum flux level and F_λ is the flux at a specific wavelength λ
- The equivalent width is essentially a measure of line strength relative to the continuum and is in units of wavelength.
- Can be used for emission lines but then W<0 and can $\rightarrow \infty$ if there is no continuum flux. For emission lines tend to use total flux in the line.

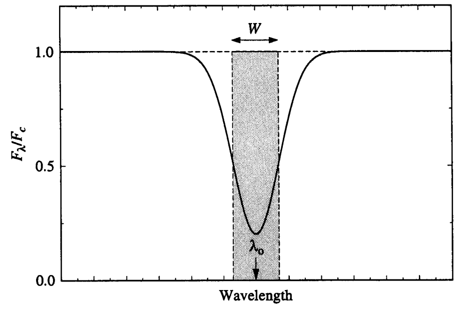

Figure 9.18 The shape of a typical spectral line.

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P Cygni profiles

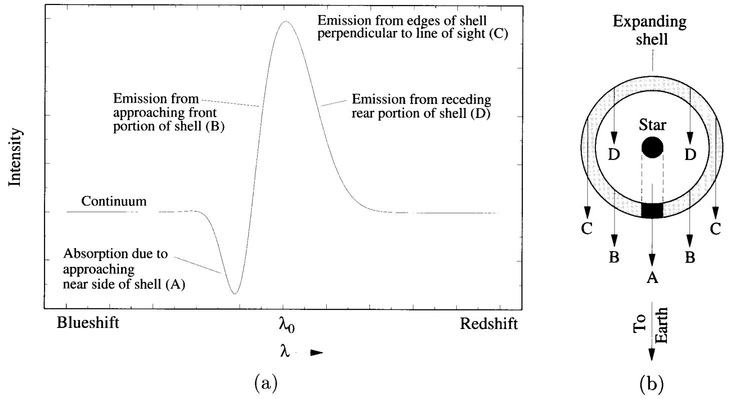

Figure 12.12 (a) A spectral line exhibiting a P Cygni profile is characterized by a broad emission peak with a superimposed blueshifted absorption trough. (b) A P Cygni profile is produced by an expanding mass shell. The emission peak is due to the outward movement of material perpendicular to the line of sight, while the blueshifted absorption feature is caused by the approaching matter in the shaded region, intercepting photons coming from the central star.

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