Part II Astrophysics/Physics

Astrophysical Fluid Dynamics
Lecture 1 : Introduction

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Administrative details

- Course materials accessible via moodle:
  - 24 lectures (recordings)
  - Slide sets for each lecture
  - PDF notes – companion to the lectures (not intended to be complete)
  - Four examples sheets

- Books:
  - Course closely follows ‘’Principles of Astrophysical Fluid Dynamics” by C.J.Clarke and R.F.Carswell
  - Other good reference books:
    - “Fluid Mechanics” (2nd Ed.) by L.D.Landau & E.M.Lifshitz
    - “The Physics of Fluids & Plasmas: An Introduction for Astrophysicists” by A.Rai Choudhuri

Today’s Lecture

- A : Basic Principles
  - What is a fluid?
  - Where do we find fluids in astrophysics?
  - Concept of a fluid element
  - Collisional and collisionless fluids

- B : Formulation of the Fluid Equations (Part I)
  - Eulerian vs Lagrangian frameworks
  - Descriptions of the kinematics (streamlines, particle paths & streaklines)

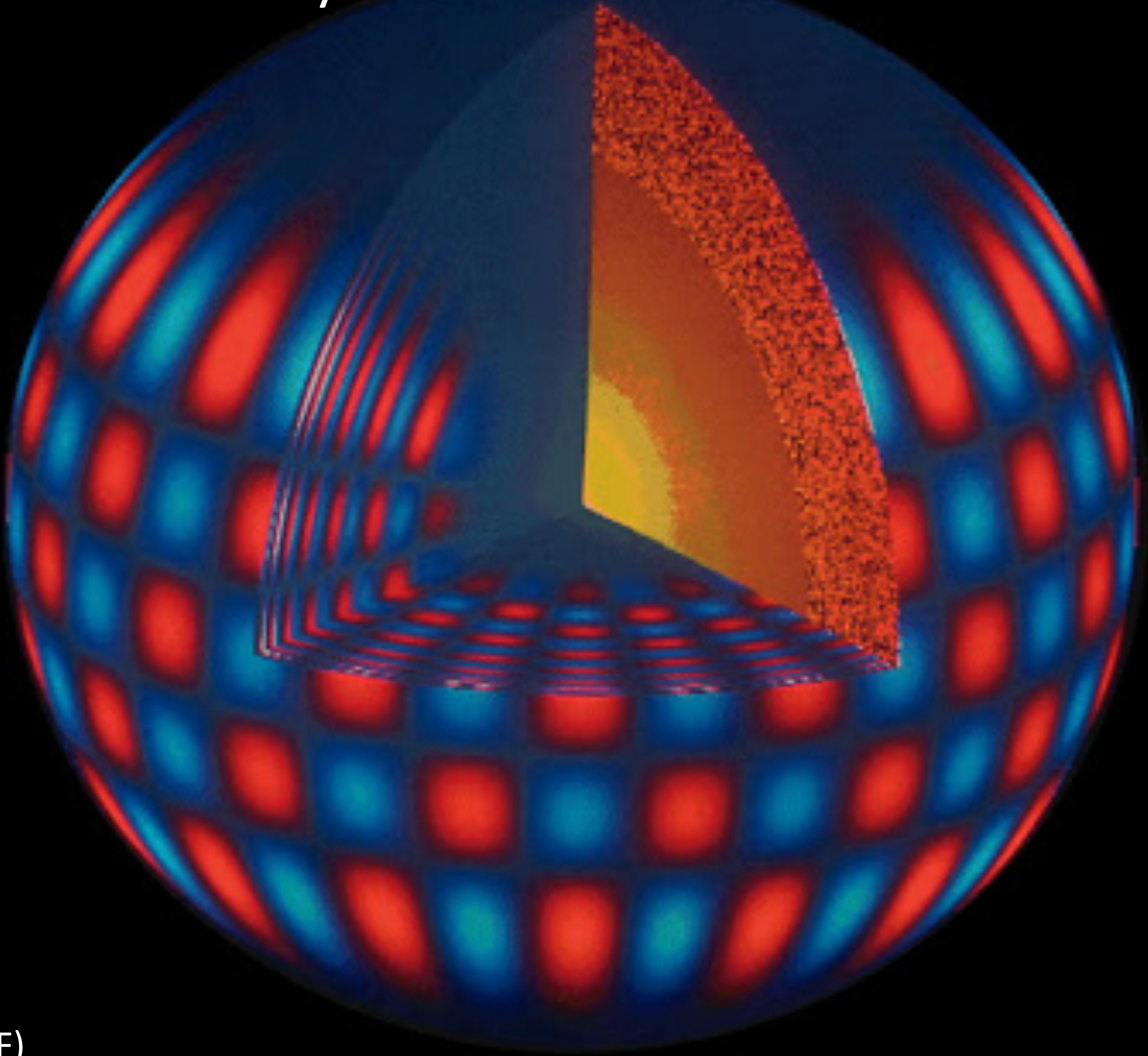
Chapter A
Basic Principles

A : Introduction

- Fluid flows!
- Fluid is described as continuous medium
  - Possesses well-defined macroscopic properties (density, velocity, pressure, …)
  - Can approach fluid dynamics from a kinetic theory perspective

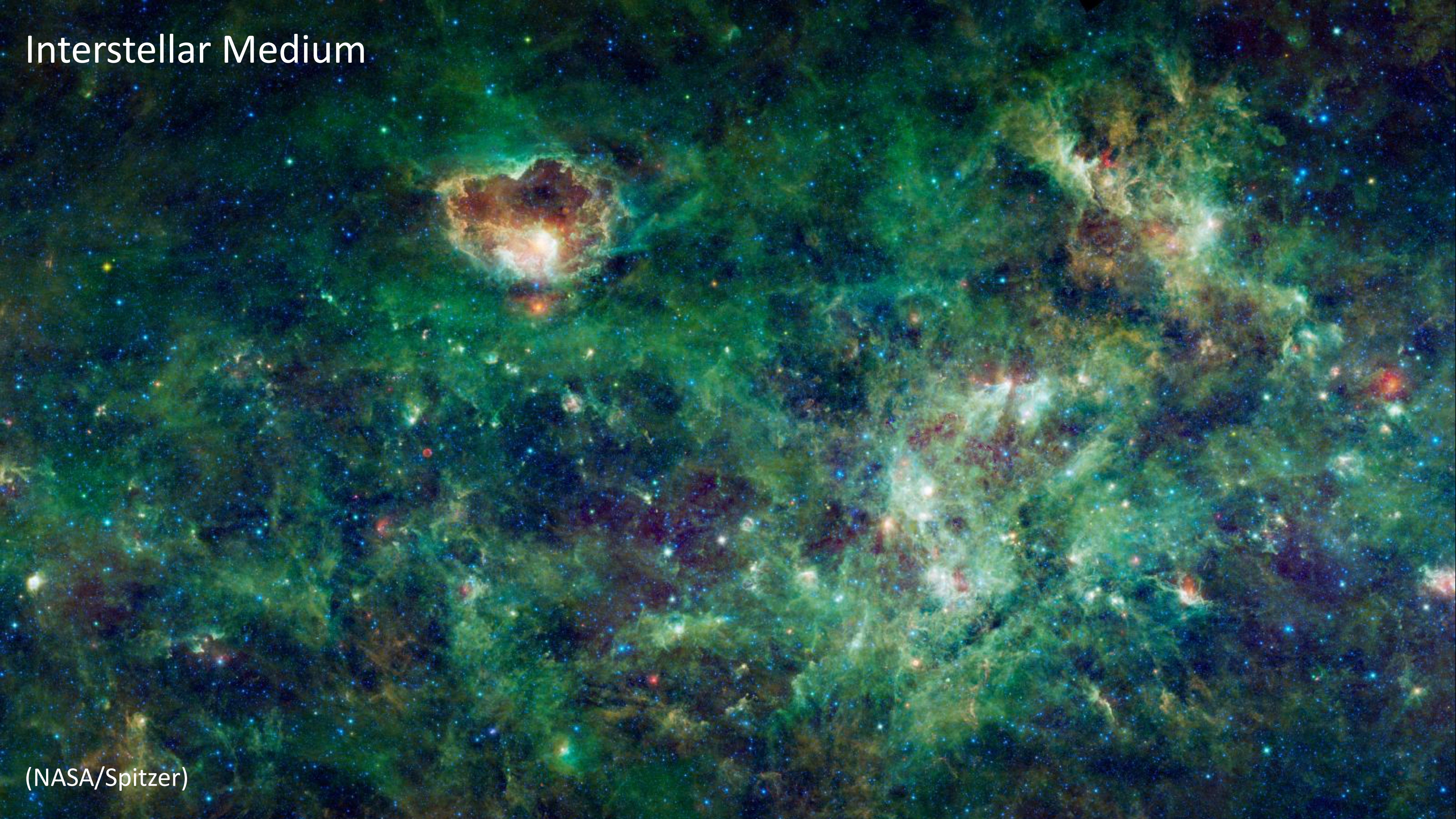
- Where do we find fluids in astrophysics?

Internal structure and dynamics of stars



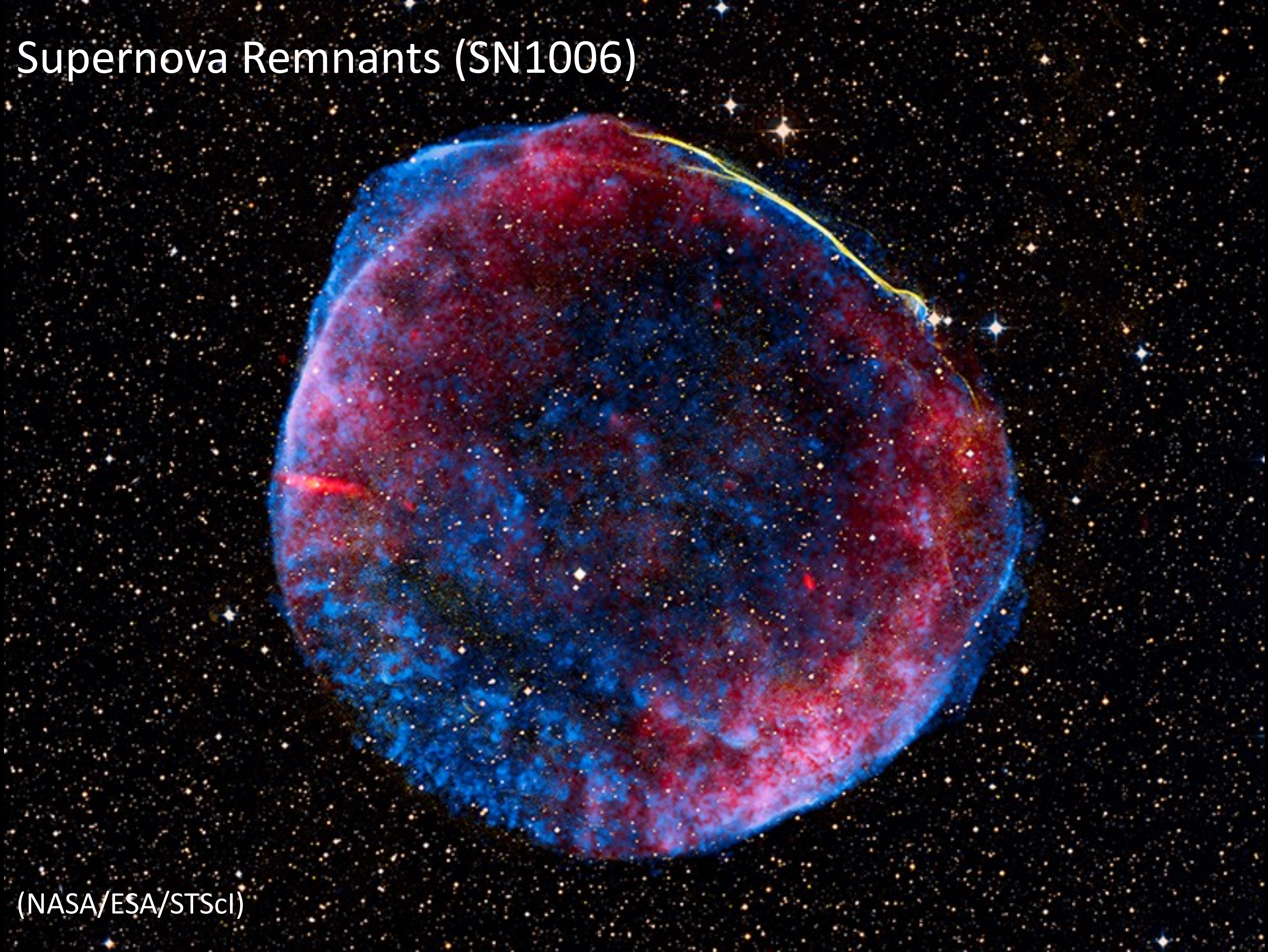
(GONG/AURA/NSF)

Interstellar Medium



(NASA/Spitzer)

Supernova Remnants (SN1006)

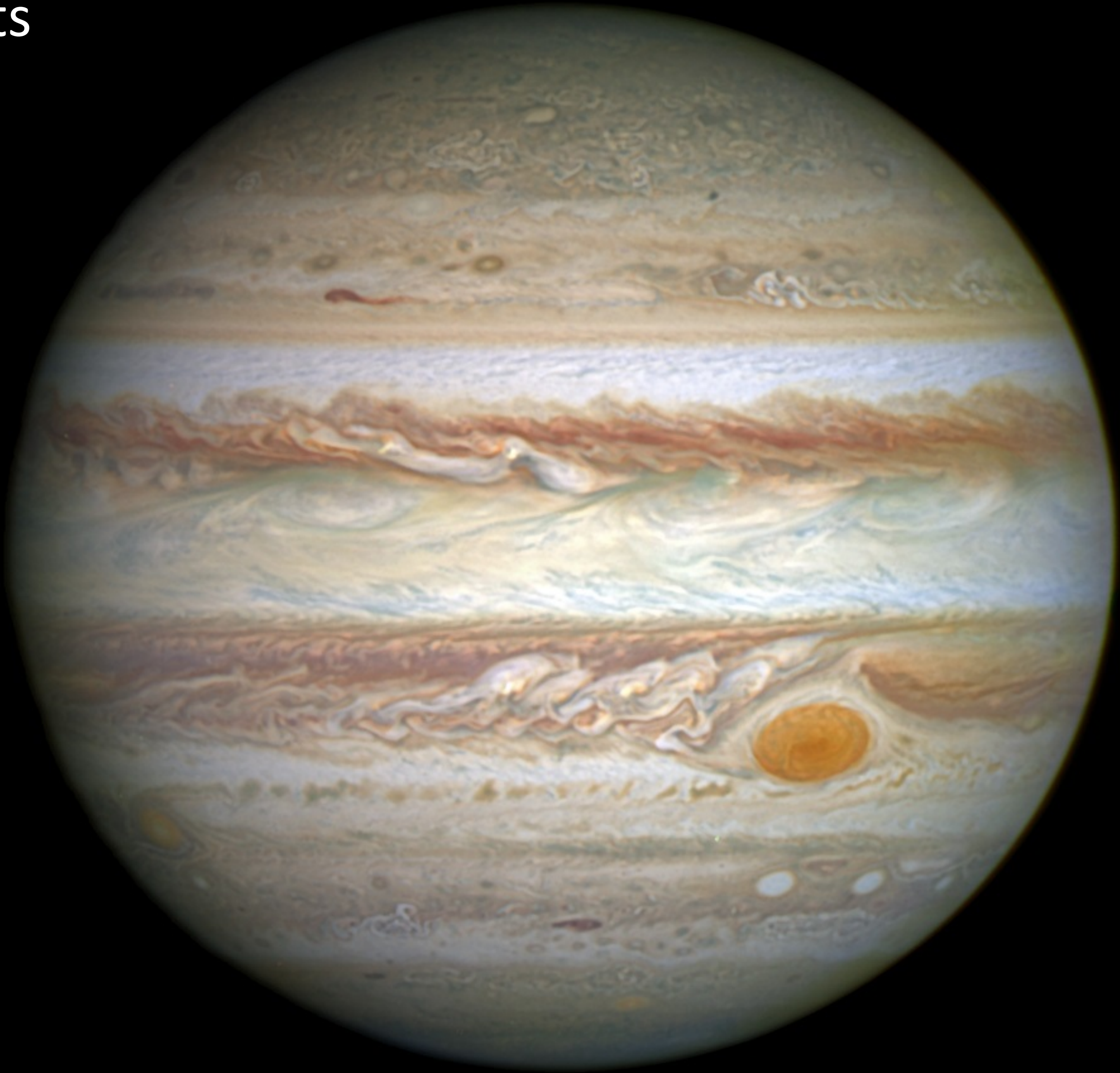


(NASA/ESA/STScI)

Accretion disks

(NASA/JPL)

Giant planets



(NASA/JPL)

Fluid element : a region of fluid (size ℓ_{region}) that is

- Small enough that every macroscopic property *q* is approx constant

$$l_{\text{region}} \ll l_{\text{scale}} \sim \frac{q}{|\nabla q|}$$

- Large enough to contain large numbers of particles

$$n l_{\text{region}}^3 \gg 1$$

(n is number of particles per unit volume)

If such fluid elements can be defined, continuum description is valid.

Otherwise, must describe the system at the particle-level.

A.2 : Collisional vs collisionless fluids

Mean-free-path λ : typical distance travelled by a particle before its direction of travel is significantly changed due to particle collisions.

In a **collisional fluid**, we have $\ell_{\text{scale}} >> \lambda$. Then…

- Particles locally attain velocity distribution that maximizes entropy.
- So, well-defined velocity distribution and pressure as function of density ρ and temperature T

$$p = p(\rho, T)$$

EQUATION OF STATE

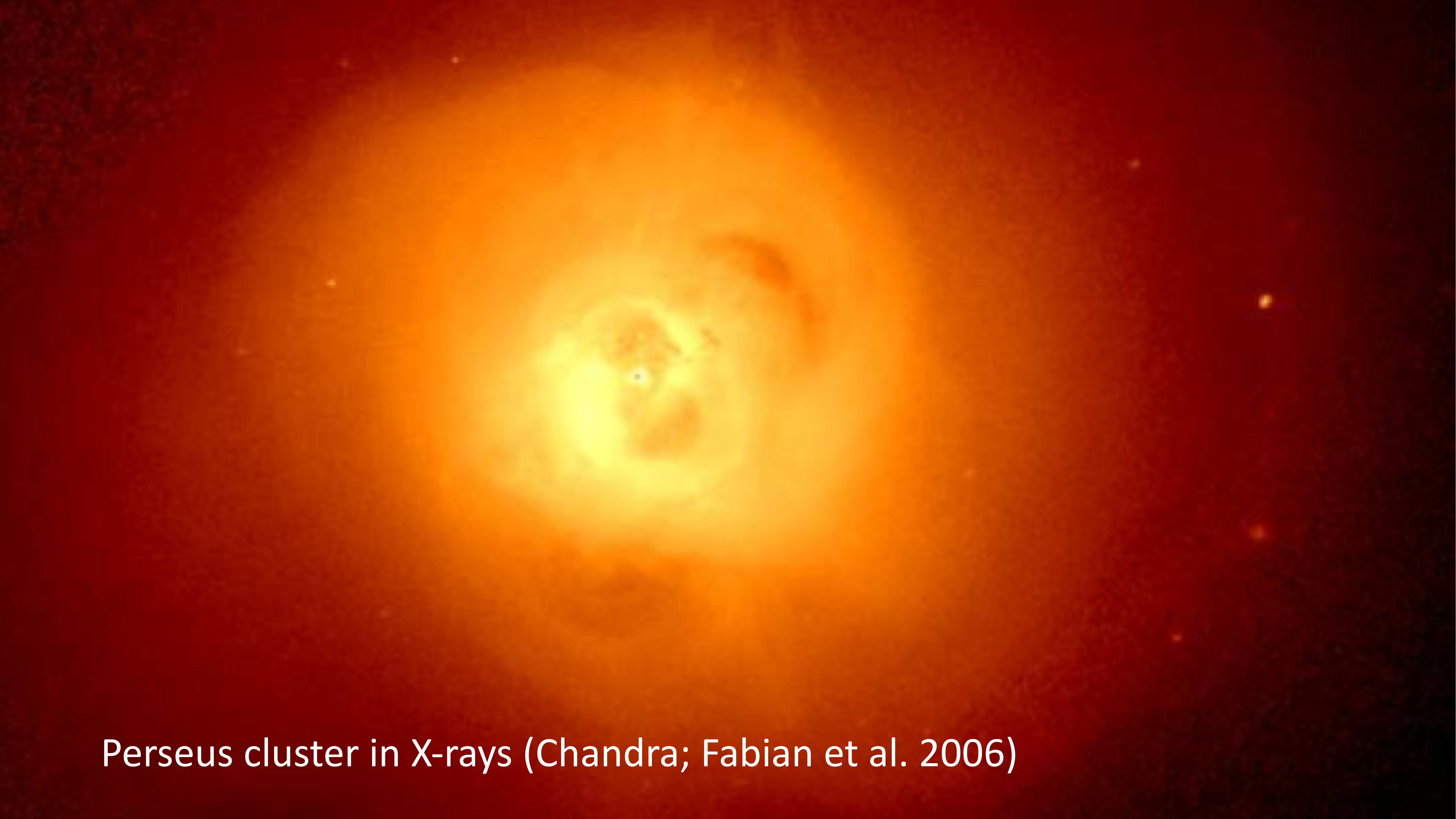
- Almost all fluids considered in this course are collisional.

- **In a collisionless fluid** $\ell_{\mathrm{scale}} << \lambda$. Then
  - Velocity distribution of particles not determined locally
  - Depends on initial conditions and non-local conditions
  - Adds significant complexity to treatment!

Examples of collisionless fluids

- ”Stellar fluid” in a galaxy
- Dark matter
- Intracluster medium of galaxy clusters (transitional case)

Perseus cluster in optical (Chandra; Fabian et al. 2006)



Perseus cluster in X-rays (Chandra; Fabian et al. 2006)

Intracluster medium : typical properties

- n_e~0.001-0.1cm $^{-3}$
- T~10^7-10^8K (fully ionized)
- R~1Mpc with ~100kpc core (1pc=3.26 light years)
- Particle-particle collisions due to Coulomb interactions

$$\lambda_e = \frac{3^{3/2}(k_B T_e)^2 \epsilon_0^2}{4\pi^{1/2} n_e e^4 \ln \Lambda}$$

$$\lambda_e = \lambda_i \approx 23 \text{ kpc} \left(\frac{T_e}{10^8 \text{ K}}\right)^2 \left(\frac{n_e}{10^{-3} \text{ cm}^{-3}}\right)^{-1}$$

Chapter B : Formulation of the Fluid Equations

B.1 : Eulerian vs Lagrangian framework

Eulerian description : consider the properties of the fluid as a function of time in a frame of reference fixed in space

$$\rho(\mathbf{r}, t), \quad p(\mathbf{r}, t), \quad T(\mathbf{r}, t), \quad \mathbf{v}(\mathbf{r}, t)$$

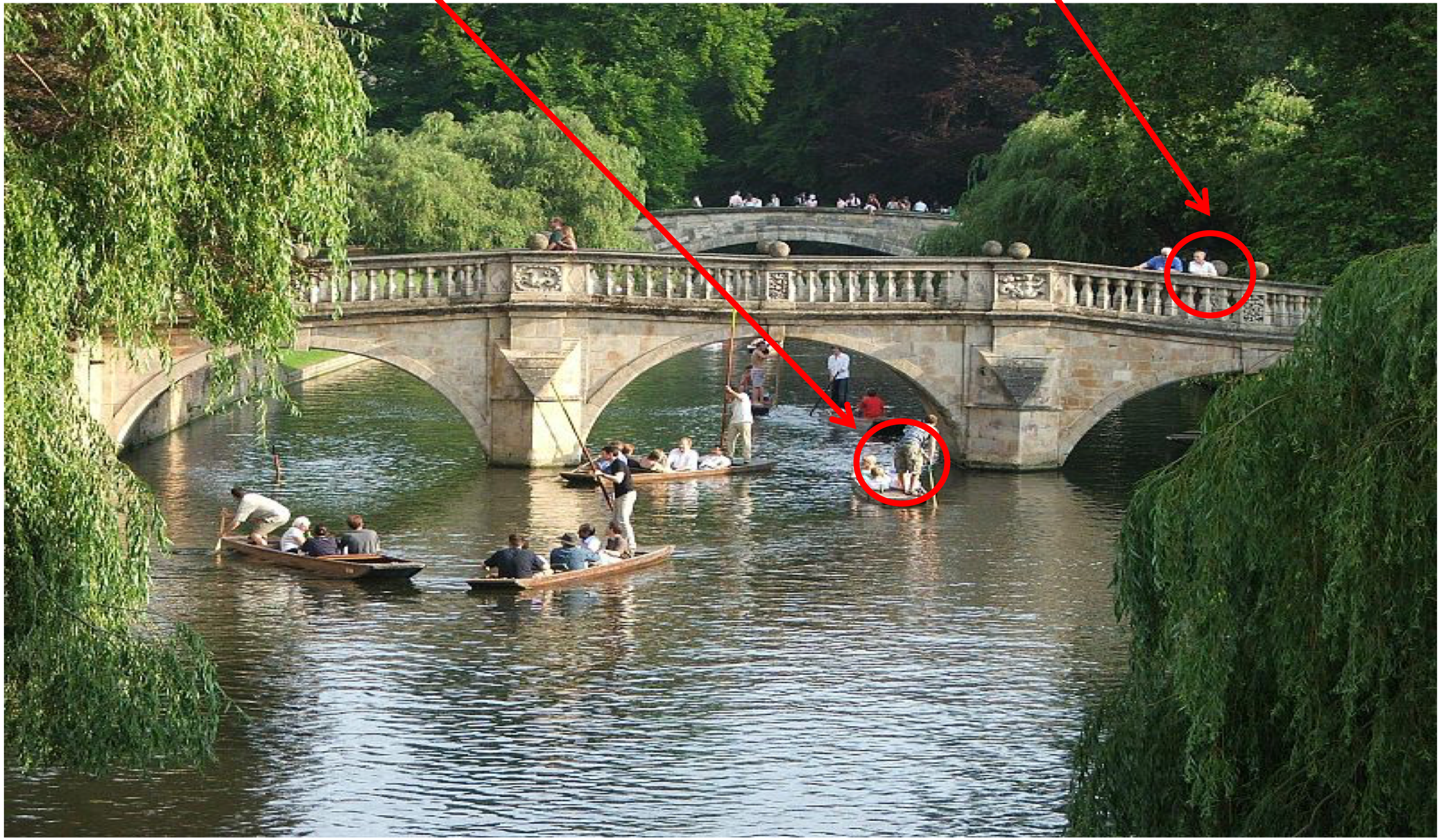
Lagrangian description : consider the properties of a particular fluid element as a function of time.

These frameworks underlie the two principle methods of computational fluid dynamics:

- Eulerian → grid-codes (space tiled by a “grid” and fluid flows through that grid)
- Lagrangian → smoothed particle codes (fluid elements treated as “smoothed particles” that propagate through continuous space.

Formulation of fluid equations

Lagrangian view **Eulerian view**



Consider fluid element with quantity Q:

- Fluid elements moves from **r** → **r** + $\boldsymbol{\delta}$**r** in time $t \rightarrow t + \delta t$
- So, rate of change of Q for fluid element is

$$\frac{\mathrm{D}Q}{\mathrm{D}t} = \lim_{\delta t \to 0} \left[\frac{Q(\mathbf{r} + \delta\mathbf{r}, t + \delta t) - Q(\mathbf{r}, t)}{\delta t} \right]$$

- But

$$Q(\mathbf{r} + \delta\mathbf{r}, t + \delta t) = Q(\mathbf{r}, t) + \frac{\partial Q}{\partial t}\delta t + \delta\mathbf{r} \cdot \boldsymbol{\nabla} Q + \mathcal{O}(\delta t^2, |\delta\mathbf{r}|^2, \delta t|\delta\mathbf{r}|)$$

- So

$$\frac{\mathrm{D}Q}{\mathrm{D}t} = \lim_{\delta t \to 0} \left[\frac{\partial Q}{\partial t} + \frac{\delta\mathbf{r}}{\delta t} \cdot \boldsymbol{\nabla} Q + \mathcal{O}(\delta t, |\delta\mathbf{r}|) \right]$$

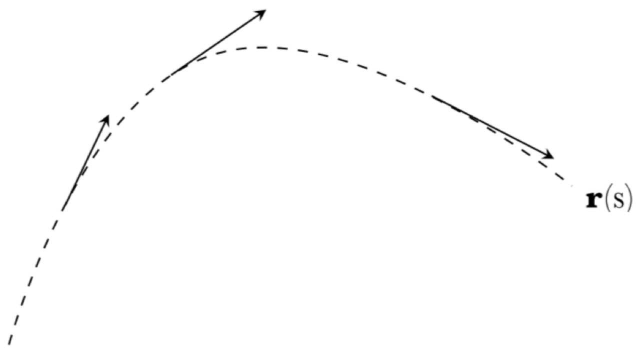
$$\underbrace{\frac{\mathrm{D}Q}{\mathrm{D}t}}_{\substack{\text{Lagrangian} \\ \text{time derivative}}} = \underbrace{\frac{\partial Q}{\partial t}}_{\substack{\text{Eulerian} \\ \text{time derivative}}} + \underbrace{\mathbf{u} \cdot \boldsymbol{\nabla} Q}_{\substack{\text{“convective”} \\ \text{derivative}}}$$

$$\mathbf{u} \cdot \nabla \equiv u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}$$

B.2 : Kinematics

Definitions:

- **Streamlines** : family of curves that are instantaneously tangent to velocity vector of the flow **u**(**r**,*t*)



$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \left(\frac{\mathrm{d}x}{\mathrm{d}s}, \frac{\mathrm{d}y}{\mathrm{d}s}, \frac{\mathrm{d}z}{\mathrm{d}s}\right)$$

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} \times \mathbf{u} = 0 \qquad \Rightarrow \qquad \frac{\mathrm{d}x}{u_x} = \frac{\mathrm{d}y}{u_y} = \frac{\mathrm{d}z}{u_z}$$

- **Particle paths** : paths through space taken by individual fluid elements; given by solution of

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{u}(\mathbf{r}, t)$$

- **Streaklines** : locus of points of all fluid elements that have passed through a given point in the past

$$\mathbf{r}(t) = \mathbf{r}_0$$

(imagine the point as a source of “dye” or ”smoke”).

Streamlines, particle paths, and streaklines all coincide if the flow is steady (i.e. $\frac{\partial \mathbf{u}}{\partial t} = 0$).

Streamlines past an airfoil