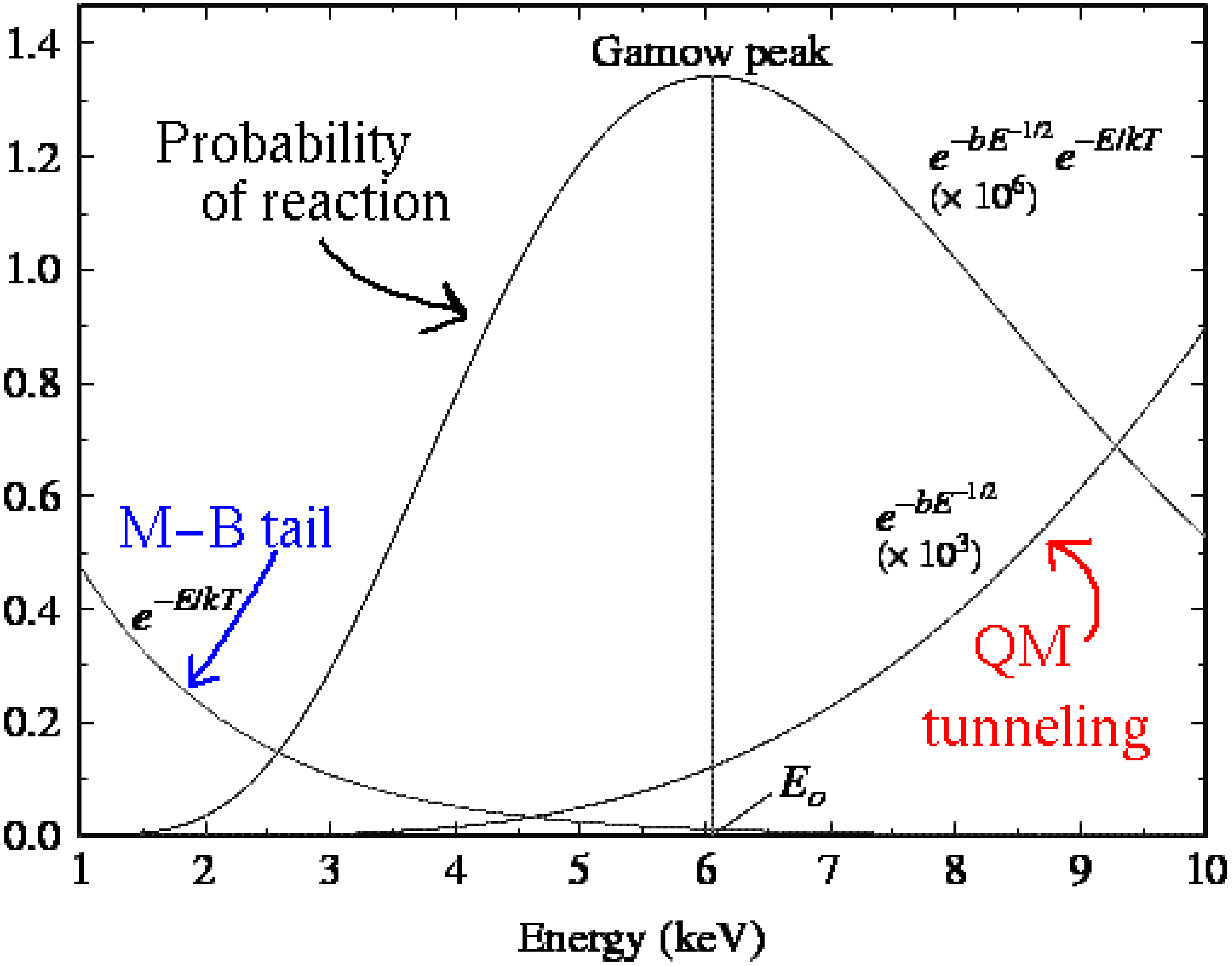
Structure and Evolution of Stars

Lecture 11: Thermonuclear Reactions (#2)

- Power-law parametrization of the nuclear energy generation rate
- Notation and conservation laws
- Hydrogen Burning
  - the p-p chain
  - the CNO cycle
- Helium Burning – triple-α process
- Carbon, Oxygen and Silicon Burning


Gamow peak
Probability of reaction
$e^{-bE^{-1/2}}e^{-E/kT}$ (× 10^6)
$e^{-bE^{-1/2}}$ (× 10^3)
M−B tail
$e^{-E/kT}$
QM tunneling
E_o
Energy (keV)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1
2
3
4
5
6
7
8
9
10

Parametrizing the Energy Generation Rate

- As with treatment of the opacity, κ, need a parametrization of the energy generation rate per unit mass, ε, as a function of composition, ρ and T
- Power-law approximation makes solution of the stellar structure equations tractable and also provides good approximations to the behaviour of ε for relevant ranges of ρ and T

Write reaction rate per unit volume, r_{ij}, as:

$$r_{ij} \cong r_0 X_i X_j \rho^{\alpha'} T^{\beta}$$

where r_0 is a constant, X_i and X_j are the mass fractions of the interaction nuclei.

- For a 2-body encounter, expect dependence on density to be ρ^2
- T-dependence turns out to have power-law indices in range 3 → 40 – which, in the extreme, means that a 10% change in T increases r_{ij} by a factor 45!

Parametrizing the Energy Generation Rate

Need the energy generation rate per unit mass, ε, which is obtained by dividing the reaction rate by the density:

$$\varepsilon_{ij} = const\,\frac{r_{ij}}{\rho}$$

The constant ε_0 depends on the energy liberated by reaction and other factors:

$$\varepsilon_{ij} = \varepsilon_0 X_i X_j \rho^{\alpha} T^{\beta}$$

have lost one power of density, i.e.:

$$\alpha = \alpha' - 1$$

Sum overall possible reaction pairs *ij* to give total rate per unit mass

$$\varepsilon = \sum \varepsilon_{ij}$$

Nuclear Reactions

- In general the fusing of nuclei takes place via a series of 2-body encounters as the probability of N-body encounters with N>2 is vanishingly small, e.g. 4 H nuclei combining together simultaneously to produce a He nucleus does not occur

- The reactions must also obey certain conservation laws
  - baryon number
  - lepton number
  - charge

- Conversion of H into He is critical as mass fraction for H is high, $X \approx 0.7$, and the unit nuclear charge means that reaction rate likely to occur at lowest T (cf nuclei with $Z>1$)

Notation

$^1_1\mathrm{H}$ subscript denotes charge

while the superscript denotes mass

thus $^2_1\mathrm{H}$ denotes a deuterium nucleus

$^4_2\mathrm{He}$ denotes a helium nucleus

e^- denotes an electron

e^+ denotes a positron

γ denotes a photon, a γ - ray in practice

ν denotes a neutrino (a lepton)

Hydrogen Burning: the p-p chain

The **p-p chain** is initiated through the interaction of two protons to form deuterium, stable isotope of H. The weak force having converted a proton into a neutron (slowest element of the p-p chain)

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu $$

Second stage involves the interaction of deuterium and H nuclei to produce a light He nucleus

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma $$

The p-p chain then involves one of two possibility, the first of which involves two light He nuclei

$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} $$

Hydrogen Burning: the p-p chain

- The 3-stage reaction has taken six H nuclei, producing a single He nucleus, two H nuclei and a photon and neutrino - known as the **p-p I chain**
- The second possibility involves the interaction of a light H nucleus and a He nucleus to produce Be
- Again, there are two possible options. In the first, capture of an electron converts a proton to a neutron, followed by interaction with a proton to produce two He nuclei – this branch is known as the **p-p II chain**

$$ {}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma $$

$$ {}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu $$

$$ {}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} $$

Hydrogen Burning: the p-p chain

- Finally, the third possibility for producing He follows if, instead of the Be nucleus capturing an electron, the Be interacts with a proton to produce Boron

$$ {}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma $$

$$ {}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma $$

The Boron then decays to produce Be, which, in turn, decays to produce two He nuclei – the third route to the production of He from H is known as the **p-p III chain**

$$ {}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu $$

$$ {}^{8}_{4}\mathrm{Be} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} $$

- All three branches of the p-p chain result in the conversion of H into He

Hydrogen Burning: the p-p chain

- All three branches of the p-p chain operate similtaneously and all three have the same overall effect
- The relative importance of the three chains depends on the *branching ratios* which determine the probability of each of the three sequences occurring
- The branching ratios depend on the density, temperature and composition of the material
- In the Sun the ratio for the split between p-p I and p-p II+p-p III combined is 0.85:0.15 – the ratio will become more equal as the mass fraction of He (Y) increases
- The ratio for the split between p-p II and p-p III is 0.999:0.001 with the large imbalance due in part to density of electrons compared to protons and the need for QM tunneling for p-p III
- Qualitatively, how will p-p II:p-p III depend on T?

Hydrogen Burning: the p-p chain

- Energy released can be calculated from the mass difference between the original four protons and the final He nucleus via $E=\Delta mc^2$, giving 26.7MeV
- Same for each of the three chains but the fraction of the energy liberated ending up as photons and neutrinos varies significantly but the average per He nucleus produced (dominated by p-p I and p-p II) is ≈26MeV
- The rate of energy release is determined by the slowest reaction in the whole sequence – the first element in the whole p-p chain which involves proton decay
- Would the lifetime of the Sun change significantly if in fact a stable bound state of He (consisting of just 2 protons) existed?

Hydrogen Burning: the p-p chain

- The p-p chain requires the lowest temperatures for the initiation of nuclear burning and is the dominant form of energy generation in low mass stars such as the Sun

- In terms of the power-law parameterisation of the energy generation rate discussed earlier:

$$\varepsilon_{p-p} \propto X^2 \rho T^4$$

- For conditions in the core of the Sun:

$$\varepsilon_{p-p} = 9.5 \times 10^{-37} X^2 \rho T^4 \ \mathrm{W m^{-3}}$$

Hydrogen Burning: the CNO cycle

- The p-p chain is not the only mechanism for fusing H to produce He

- Although we have discussed that the mass fraction of metals is small ($Z \approx 0.02$) and thus the density of Carbon, Nitogen and Oxygen is small relative to H, the presence of C, N and O mediates the fusing of H into He by acting as catalysts

- There are two branches to the **CNO cycle** both involving six reactions that produce one He nucleus via four proton captures, two beta decays and emission of two neutrinos

Hydrogen Burning: the CNO cycle

The first, and dominant branch, proceeds via these six reactions

Note how the C, N and O abundances are unchanged at the completion of the sequence

$$ {}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma $$

$$ {}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu $$

$$ {}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma $$

$$ {}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma $$

$$ {}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^{+} + \nu $$

$$ {}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} $$

Hydrogen Burning: the CNO cycle

The second, and less common sequence, also involves six reactions following a branch in the first sequence when N plus a proton produces an O nucleus

Again, note how the N, O and F abundances are unchanged at the completion of the sequence

$$^{14}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \rightarrow ^{15}_{8}\mathrm{O} + \gamma$$

$$^{15}_{8}\mathrm{O} \rightarrow ^{15}_{7}\mathrm{N} + e^{+} + \nu$$

$$^{15}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \rightarrow ^{16}_{8}\mathrm{O} + \gamma$$

$$^{16}_{8}\mathrm{O} + ^{1}_{1}\mathrm{H} \rightarrow ^{17}_{9}\mathrm{F} + \gamma$$

$$^{17}_{9}\mathrm{F} \rightarrow ^{17}_{8}\mathrm{O} + e^{+} + \nu$$

$$^{17}_{8}\mathrm{O} + ^{1}_{1}\mathrm{H} \rightarrow ^{14}_{7}\mathrm{N} + ^{4}_{2}\mathrm{He}$$

Hydrogen Burning: the CNO cycle

- For a substantial range of (high) temperatures the capture reactions with the protons are the slowest elements of the cycles and thus there is a very strong dependence of the reaction rate on *T*

$$\varepsilon_{CNO} \propto X X_{CNO} \rho T^{18}$$

- Energy generation rate per He nucleus created is very similar to that for the p-p chain

- CNO cycle dominates hydrogen burning in stars with masses >1.5 Solar masses

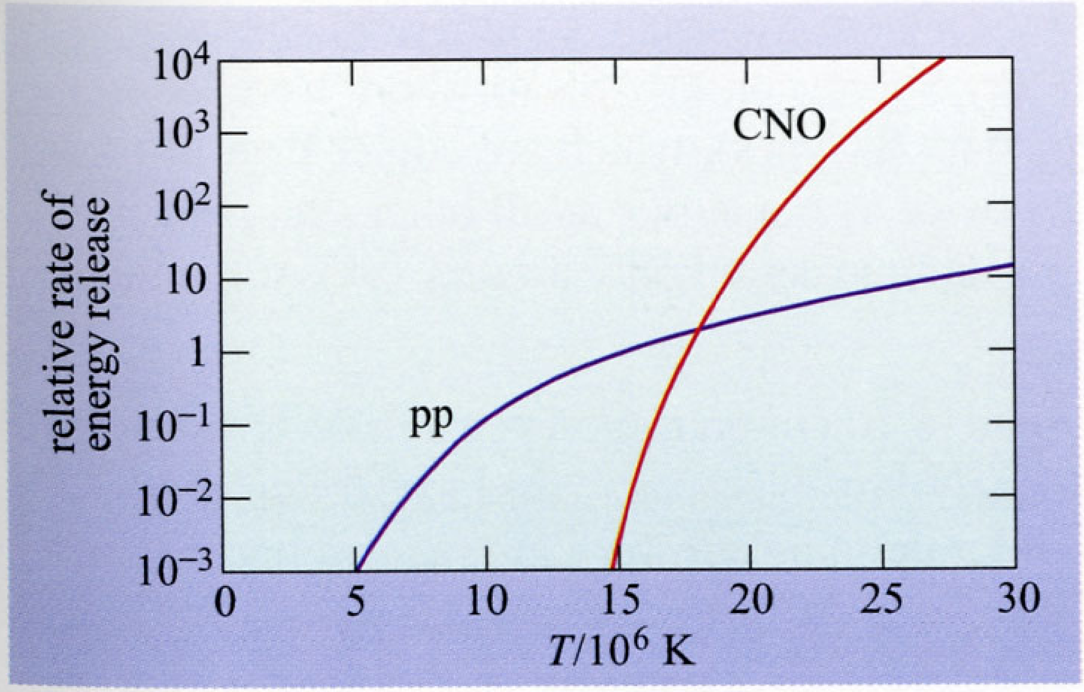


Figure 6.9 The rate of energy release for the three pp and CNO reaction chains as a function of temperature. A relative abundance of the elements as for the Sun has been assumed.

Helium Burning: the triple-α reaction

- Note that the increase in binding energy per nucleon is not a smooth function of atomic number
- Particularly stable nuclei with equal and even numbers of protons and neutrons are unusually stable
- Helium is such a nucleus and to fuse He and H to produce Li *requires* energy, i.e. *endothermic*
- Two He nuclei can fuse to produce Beryllium-8 (4 protons and 4 neutrons) but the isotope is not stable
- Our fusion process is therefore stuck at He
- Solution due to Ed Salpeter, who realised that at sufficiently high T, $\approx 10^8$K, the probability of an encounter with another He nucleus to produce Carbon before decay is significant. Thus the creation of C is possible via what is effectively a 3-body interaction

Binding Energy per Nucleon

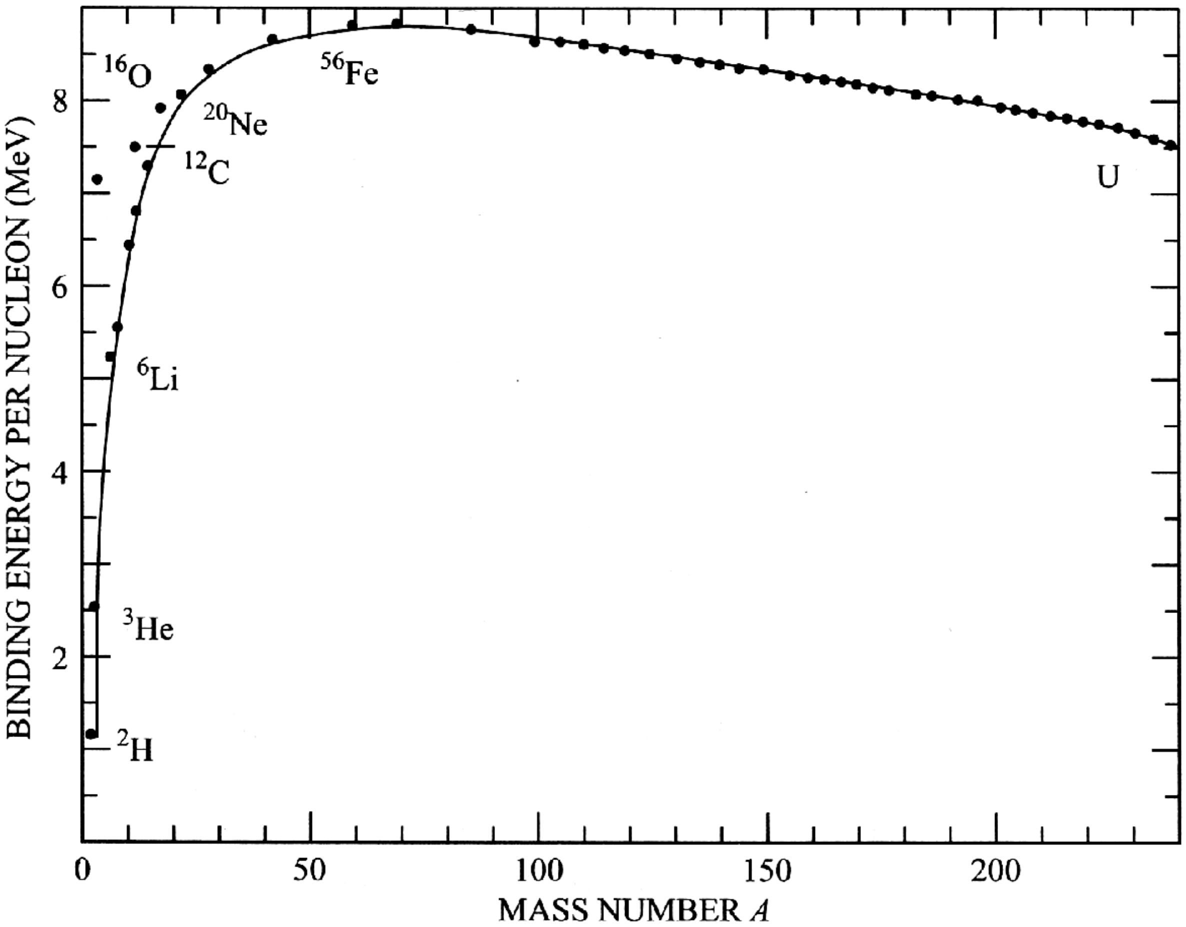


Fig. 1.3 Binding energy per nucleon for atomic nuclei. There is a broad maximum at mass number 56 which implies that energy is normally released when two light nuclei fuse to form a heavier nucleus provided the nucleus formed has a mass number less than 56

Helium Burning: the triple-α reaction

- Fred Hoyle then calculated that the fusion of Beryllium and Helium was a resonant reaction, producing a dramatic increase in the predicted reaction rate

$$^{4}_{2}\mathrm{He}+^{4}_{2}\mathrm{He}\leftrightarrow ^{8}_{4}\mathrm{Be}$$

$$^{8}_{4}\mathrm{Be}+^{4}_{2}\mathrm{He}\rightarrow ^{12}_{6}\mathrm{C}+\gamma$$

- The 3-body nature of the reaction leads to extraordinary dependence on *T* and also note effect on the mass-fraction and density dependence

$$\varepsilon_{3\alpha} \propto Y^{3}\rho^{2}T^{40}$$

Helium Capture

- With sufficient mass-fractions of He and higher mass nuclei it is possible to create higher mass nuclei by He capture. However, the increasing height of the Coulomb barrier means that only He capture by C is significant at the temperatures when the triple-α process is important

$$ {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma $$

- At higher *T* still the O and He can fuse to form neon

$$ {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma $$

- Temperatures of several $\times 10^{8}$K necessary and energy available compared to that available from hydrogen burning small

High Temperature Fusion

- Once $T \approx 6\times10^8$K then Carbon fusion can take place, producing a range of different products, including Ne, Na and Mg

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C}+{}^{12}_{6}\mathrm{C} &\rightarrow {}^{16}_{8}\mathrm{O}+{}^{4}_{2}\mathrm{He}+{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{20}_{10}\mathrm{Ne}+{}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{23}_{11}\mathrm{Na}+{}^{1}_{1}\mathrm{H} \\
&\rightarrow {}^{23}_{12}\mathrm{Mg}+n \\
&\rightarrow {}^{24}_{12}\mathrm{Mg}+\gamma
\end{aligned}
$$

High Temperature Fusion

- Once $T \approx 10^9$K then Oxygen fusion can take place, producing a range of different products, including Mg, Si, P and S

$$
\begin{aligned}
{}^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} &\rightarrow {}^{24}_{12}\mathrm{Mg} + {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} \\
&\rightarrow {}^{31}_{15}\mathrm{P} + {}^{1}_{1}\mathrm{H} \\
&\rightarrow {}^{31}_{16}\mathrm{S} + n \\
&\rightarrow {}^{32}_{16}\mathrm{S} + \gamma
\end{aligned}
$$

High Temperature Fusion

- As T reaches $T \approx 2\times10^9$K Silicon burning can occur, producing nuclei up to the iron peak. However, by now the energy of the photons is so high that nuclei are subject to **photodisintegration** breaking down into nuclei of lower mass with many He nuclei produced

- Interplay between Si burning and photodisintegration continues with build-up of the most stable nuclei – Fe, Co, Ni – which are most resistant to photodisintegration until $T \approx 7\times10^9$K

Major Nuclear Burning Processes

Nuclear Fuel	Process	T (10^6K)	Products	Energy (per nucleon MeV)
H	p-p	~4	He	6.55
H	CNO	15	He	6.25
He	Triple-α	100	C,O	0.61
C	C+C	600	O,Ne,Na,Mg	0.54
O	O+O	1000	Mg,S,P,Si	~0.3
Si		3000	Co,Fe,Ni	~0.18

From Prialnik p68

Lecture 11: Summary

- Hydrogen burning provides principal energy generation for stars via p-p chain and CNO cycle
- Triple-α process at temperatures $T \approx 10^8$K provide mechanism for overcoming local peak in the binding energy per nucleon for Helium
- At increasingly high *T* the energies of the nuclei are sufficient to overcome the increasingly high Coulomb barriers associated with heavy nuclei, with C, O and Si burning
- At extremely high temperatures, $T \approx 5 \times 10^9$K, photodisintegration of heavy nuclei becomes important
- Limit to energy generation by fusion limited first by photodisintegration and fundamentally by burning products reaching iron peak in the binding energy per nucleon

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