CHINE:
a new code for accurate massive BH dynamics in galaxy formation simulations

Collaborators:
Sverre Aarseth (IoA)
Martin Haehnelt (IoA/Kavli)
Thorsten Naab (MPA Garching)
Rainer Spurzem (NAOC/CAS Beijing)

Simon Karl - IoA/Kavli Cambridge

“Mind the Gap:
from microphysics to large-scale structure in the Universe”
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Massive black holes in galaxy centers

Super-massive black holes with masses of $10^6 - 10^{10} \, M_\odot$ are commonly observed in the centers of nearby galaxies (Kormendy & Richstone 1995, Ferrarese & Ford 2005).

... best example being the Milky Way (Ghez et al. 2008, Gillessen et al. 2009).
Evidence for massive BHs in high redshift quasars (Fan et al. 2001, Willott et al. 2010).

Quasi-parallel evolution of BH accretion history and star formation history (Hasinger et al. 2005, Madau et al. 1998).

Tight scaling relations between BH masses and host galaxy properties: $M_{\text{BH}}$-$\sigma / M_{\text{BH}}$-$M_{\text{Bulge}} / M_{\text{BH}}$-$L_{\text{Bulge}} / M_{\text{BH}}$-$N_{\text{GC}}$ / etc. (Kormendy & Richstone 1995, Ferrarese & Merritt 2000, Tremaine et al. 2002, Häring & Rix 2004, etc.).
Massive BH - galaxy co-evolution

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Tight scaling relations between BH masses and host galaxy properties: \( M_{BH} - \sigma / M_{BH} - M_{Bulge} / M_{BH} - L_{Bulge} / M_{BH} - N_{GC} / etc. \) (Kormendy & Richstone 1995, Ferrarese & Merritt 2000, Tremaine et al. 2002, Häring & Rix 2004, etc.).

Gültekin et al. 2009
Mind the gap!

How do black holes evolve with/within their hosts over cosmic time?

For accurate (dynamical!) modeling of massive BH (multiples) evolution in whole galaxies /mergers we need to resolve many orders of magnitude in:
- time: \(~\text{Gyr} \leftrightarrow \text{yr}\)
- space: \(~100 \text{ kpc} \leftrightarrow 1 \text{ mpc}\)
- mass \(~10^{12} \text{ M}_\odot \leftrightarrow 1 \text{ M}_\odot\)

Not possible with current numerical tools!!!

Problem:

<table>
<thead>
<tr>
<th>Tree-Codes / Mesh-codes:</th>
<th>Direct N-body codes</th>
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<tbody>
<tr>
<td>• limits in accuracy (e.g. leapfrog)</td>
<td>• good accuracy &amp; resolution</td>
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<tr>
<td>• softening/CIC mimics collisionless systems</td>
<td>• limited in particle number</td>
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<tr>
<td>• (typically) poor mass resolution</td>
<td>• do not include gas dynamics</td>
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... construct new hybrid code:


- algorithmic regularization (AR) scheme* for particles near a massive BH (Preto & Tremaine 1999, Mikkola & Tanikawa 1999a/b, Mikkola & Aarseth 2002)

**Goal:** Compute sub-system subject to strong gravitational interactions with high accuracy, while the rest of the galaxy is still evolved with the tree-code!

* c.f. similar implementations in N-body codes (Harfst et al. 2008, Nitadori & Aarseth 2012)
Classical Regularization

‘Levi-Civita’ transformation of 2-body coordinates and time in 1D:

**Singular Hamiltonian & EoM**

\[ H = \frac{p^2}{2\mu} - G\frac{\mu M}{r} = E \]

**EoM:**

\[ \frac{d^2 r}{dt^2} + \frac{GM}{r^2} = 0 \]

**Transformed Hamiltonian & regular EoM**

\[ H = \frac{P^2}{8u^2\mu} - G\frac{\mu M}{u^2} = E \]

**EoM:**

\[ \frac{d^2 u}{ds^2} - \frac{E}{2\mu} u = 0 \]

\[ r = u \cdot u \Rightarrow P = 2u \cdot p \]

\[ dt = r \, ds \]

History: 2D: Levi-Civita 1920  
3D: not possible; via 4D by Kustaanheimo & Stiefel 1965  
> 2 bodies: Mikkola & Aarseth 1989
Algorithmic Regularization*

- apply *time transformation (only)* to Hamiltonian of Newtonian few-body system (Mikkola & Aarseth 2002)
- evolve coordinates & (original) time + their ‘momenta’ by means of simple leapfrog
- high accuracy given by Bulirsch-Stoer extrapolation method (Gregg 1965, Bulirsch & Stoer 1966)
- use chain concept of smallest inter-particle vectors (Mikkola & Aarseth 1990) to reduce round-off errors => AR-chain

*strictly speaking no (classical) regularisation, but obtains regular results*
CHINE in a nutshell

• both parent codes written in Fortran
• new interface between the two codes
• particles near massive body become member of the AR-chain; integration in local CoM reference frame
• CoM of AR-chain is evolved as single particle in the tree-code
• after every chain time-step: absorption to and escape from chain & neighbour search for nearest particles (‘perturbers’)
• treatment of additional external forces on chain particles due to perturbers
• corrections to ‘CoM particle’ force by resolving perturber forces onto chain particles
• direct force calculations for perturbers due to chain particles
Particle orbits
Model:

Hernquist sphere
(a.k.a. Dehnen 1993, Tremain et al. 1994 models with $\eta = 2$)
with $N = 10^5$ particles

$M = r_{\text{sys}} = (G =)1$
$\Rightarrow r_{\text{hmr}} = 2.41$
$m_\star = 10^{-5}$
$\epsilon_\star = 0.0 / 0.02$

1 BH of $m_{\text{BH}} = 0.1\%M$
set @ $r_0 = r_{\text{hmr}}$
with $v_0 = v_{\text{circ}}(r_0)$
Dynamical friction comparison

\[
a_{DF} = -4\pi G^2 \frac{M_{BH}}{v_{BH}^2} \rho(v_\star < v_{BH}) \ln \Lambda
\]

(Binney & Tremaine 1987)

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$$\ln \Lambda = \ln \left( \frac{b_{\text{max}}}{\sqrt{b_{\text{min}}^2 + a_{90}^2}} \right)$$

$$b_{\text{max}} = \frac{\rho}{|\nabla \rho|} = \frac{R}{3 - \eta}$$

$$(\text{Just et al. 2011})$$

$$a_{90} \approx \frac{G M_{\text{BH}}}{2\sigma^2 + v_{\text{BH}}^2}$$

$$(\text{Binney & Tremaine 1987})$$
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\rho(v_\star < v_{\text{BH}}) = \kappa_s \cdot \rho_s(v_\star < v_{\text{BH}})
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\varepsilon_{\text{max}} = 1.5 \varepsilon
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Nbody6

VINE - $\varepsilon_{BH}=0.1$

VINE - $\varepsilon_{BH}=\varepsilon_{\star}$
Dynamical friction comparison

Nbody6

CHINE

VINE - $\varepsilon_{BH} = \varepsilon_{\star}$
Conclusions

...with an AR-chain method it seems to be possible to jump the gap (posed by grav. softening) form above!

What’s next ...

• extend to multiple chains
  -> run high-resolution (‘dry’) merger simulations including SMBHs

• include more physics:
  - SPH -> effects of gas drag on the SMBHs
  - post-newtonian relativistic terms
    -> accurate evolution of final binary decay & timing of merger

• MPI parallelization...

• ...or, port to other tree codes, moving mesh/AMR grav. solvers
  -> cosmo re-sims; BH demographics, etc...
THANK YOU!!