Secular evolution of planetary eccentricity during migration

Enrico Ragusa, Giuseppe Lodato – Università degli Studi di Milano
Giovanni Rosotti, Richard Booth, Cathie Clarke – IoA
Jean Teyssandier – DAMTP
Motivation

Why eccentricity in migration workshop?

Eccentricity is the result of planet-disc exchange of energy and angular momentum, same as migration.

Eccentricity evolution and migration are two sides of the same coin.
Introduction
Motivation

- Why eccentricity in migration workshop?

- Eccentricity is the result of planet-disc exchange of energy and angular momentum, same as migration

- Eccentricity evolution and migration are two sides of the same coin
Introduction
What causes eccentricity growth?

- Mean eccentricity in exoplanets (EOD 2017): $\langle e \rangle \sim 0.06$

- Hot-Jupiters:
  - High level of eccentricity during migration
  - Blue squares $e > 0.1$

- Two main mechanisms to excite eccentricity:
  - After disc dispersal, gas poor environment: Interaction of planets with other massive bodies in the system (Rasio & Ford 1996)
  - In protoplanetary disc, gas rich environment: planet-disc interaction provides migration and eccentricity excitation (Kley & Nelson 2012, Review)
Introduction
Planet-disc resonant interaction

- Interaction at resonant locations:
  - Lindblad resonances: pump eccentricity
  - Corotation resonances: damp eccentricity
- Planets embedded in the disc: eccentricity damping (Cresswell et al. 2007, Bitsch & Kley 2010)
- Planets carving cavities ($M_p \gtrsim M_j$): saturation of coorbital torque (Ogilvie & Lubow 2003, Goldreich & Sari 2003) $e_{max} \sim 0.1$
- Important: mutual interaction, eccentricity grows also in the disc
Numerical Simulations
Planet-disc secular interaction

- Often neglected but still important secular interaction:
  - Provides periodic exchange of eccentricity between disc and planet
- Well developed in the context of celestial mechanics
- AMD (Angular Momentum Deficit) conserved

\[ AMD_p = M_p \left( \sqrt{GM_\ast a_p} - h_p \right) \approx \frac{1}{2} e_p^2 M_p \sqrt{GM_\ast a_p} \]
Numerical Simulations
Long duration numerical simulations

- FARGO3D in 2D configuration (Rosotti et al. 2017)
  \[ M_p = 13 M_J \]
- Two different disc masses \( q = \frac{M_d}{M_p} \)
  - \( q = 1/5 \), light (Rosotti et al. 2016)
  - \( q = 3/5 \), massive
- Viscous time \( \tau_v = 1.2 \times 10^5 t_{\text{orb}} \)
- Number of orbits \( N_{\text{orb}} = 1.6 \times 10^5 t_{\text{orb}} \)
Numerical Simulations
Results: Eccentricity Evolution

- Rapid initial growth of disc eccentricity (both cases)
- Periodic oscillations superimposed to eccentricity damping/pumping
Numerical Simulations
Results: Pericenter phase

- Precession of the pericenter phase
- Initial anti-alignment in both cases, then just in the massive case
Numerical Simulations
Results: Pericenter phase

- Precession of the pericenter phase
- Initial anti-alignment in both cases, then just in the massive case
Numerical Simulations
Results: Pericenter phase

- Precession of the pericenter phase
- Initial anti-alignment in both cases, then just in the massive case
Interpretation of the results
Can we treat the disc as a planet?

- Apparently yes! The disc behaves rigidly... (Teyssandier & Oglivie 2016)
Interpretation of the results
Can we treat the disc as a planet?

- Equations well developed for the three body problem in celestial mechanics (Murray & Dermott, 2001).

\[
\begin{pmatrix}
\dot{E}_p \\
\dot{E}_d
\end{pmatrix} = i \Omega_{sec} \begin{pmatrix}
q & -q \beta \\
-\sqrt{\alpha} \beta & \sqrt{\alpha}
\end{pmatrix} \cdot \begin{pmatrix}
E_p \\
E_d
\end{pmatrix}
\]

- Solution: linear combination of two modes

Slow Mode, pericenters aligned

\[
\begin{pmatrix}
E_p(t) \\
E_d(t)
\end{pmatrix} = C_1 \begin{pmatrix}
\eta_s \\
1
\end{pmatrix} e^{i \omega_s t} + C_2 \begin{pmatrix}
\eta_f \\
1
\end{pmatrix} e^{i \omega_f t}
\]

Fast mode, pericenters antialigned

\[
\begin{align*}
\left| E_j \right| e^{i \Phi_j} & = \frac{b_{3/2}^2(\alpha)}{b_{3/2}^2(\alpha)} \\
\alpha & = \frac{a_p}{a_d} \\
q & = \frac{M_d}{M_p} \\
\Omega_{sec} & = \frac{1}{4} \Omega_p \frac{M_p}{M_*} \alpha^2 \frac{b_{3/2}^2(\alpha)}{b_{3/2}^2(\alpha)}
\end{align*}
\]
Interpretation of the results

Eccentricity evolution

- It follows that eccentricity reads

\[ |E_p| = \left[ C_1^2 \eta_s^2 + C_2^2 \eta_f^2 + 2C_1 C_2 \eta_s \eta_f \cos(\Delta \varpi) \right]^{1/2}, \]

\[ |E_d| = \left[ C_1^2 + C_2^2 + 2C_1 C_2 \cos(\Delta \varpi) \right]^{1/2} \]

\[ \Delta \varpi = (\varpi_f - \varpi_s) \]

- Intensity of planet oscillations wrt disc ones

\[ \frac{\Delta |E_p|^2}{\Delta |E_d|^2} \propto \eta_s \eta_f \propto \frac{q}{\sqrt{\alpha}} < 1 \rightarrow \frac{q}{\sqrt{\alpha}} \propto \frac{J_d}{J_p} \]
Interpretation of the results

Pericenter phase

- Light case: fast antialigned mode rapidly disappears (~40000 orbits), moving toward slow aligned mode.
- Massive case: fast antialigned mode

Extraction of parameters:

\[ \alpha^{-1} \approx 4.5 - 5.0 \]

\[ q \approx 0.08 \]

Only a fraction of the disc contributes! Good scaling:

\[ \frac{q}{\sqrt{\alpha}} = \frac{\epsilon J_d}{J_p}, \quad \epsilon < 1 \]

\[ M_{d,m}/M_{d,l} = 3 \]
Interpretation of the results
Pericenter phase

- Light case: fast antialigned mode rapidly disappears (~40000 orbits), moving toward slow aligned mode.
- Massive case: fast antialigned mode

Evolution of mode intensity during the simulation

\[ \alpha^{-1} \sim 4.5 - 5.0 \]
\[ q \sim 0.08 \]
\[ q \sim 0.24 \]

Only a fraction of the disc contributes!
Good scaling:

\[ q = \frac{\epsilon J_d}{J_p}, \quad \epsilon < 1 \]

\[ M_{d,m}/M_{d,l} = 3 \]
A simple toy model
Addition of resonant and viscous terms

- Resonances acts pumping eccentricity
- Viscosity damps disc eccentricity

- **Simplest** toy model adding diagonal imaginary terms

\[
\begin{pmatrix}
\dot{E}_p \\
\dot{E}_d 
\end{pmatrix} = i\Omega_{\text{sec}} \begin{pmatrix}
q - i\lambda_1 & -q\beta \\
-\sqrt{\alpha}\beta & \sqrt{\alpha} + i\lambda_2 
\end{pmatrix} \cdot \begin{pmatrix}
E_p \\
E_d 
\end{pmatrix}
\]

- Resulting eigenvalues of the type

\[
g_{s,f} = \omega_{s,f} + i\gamma_{s,f}
\]

- Presence of imaginary part provides exponential growth/decay of the modes

Zhang et al. (2013)
A simple toy model
Addition of resonant and viscous terms

- Resonances acts pumping eccentricity
- Viscosity damps disc eccentricity

- Expected damping of fast mode (weaker)
- Higher q predicts a slower pumping and damping, consistent with simulations
- Assuming thus a maximum level of disc eccentricity, we might expect
  \[ e_{p,\text{max}} = \eta e_{d,\text{max}}, \quad e_{d,\text{max}} \sim 0.15 \]
  \[ e_{p,\text{max}} \lesssim 0.5 \]
- Aligned if \( J_d < J_p \), antialigned if \( J_d > J_p \)
- In principle at disc dispersal we are always in the first case
A simple toy model
Time evolution of the model parameters

- As time passes: disc mass decreases, planet migrates, and viscous spreading moves the center of mass of the disc.

- Evolution of eigen-frequencies
  \[
  \frac{J_d}{J_p} \propto \frac{q}{\sqrt{\alpha}}
  \]

- Beat frequency increases in light case and decreases in massive one

- Not easy to disentangle $q$ from $\alpha$
Conclusions

- Disc-planet interaction excite eccentricities up to >0.1, smaller q→ higher max eccentricities
- Ratio of angular momenta determines the evolution $\frac{q}{\sqrt{\alpha}} \propto \frac{J_d}{J_p}$
- Planets may undergo eccentricity pumping at late times, even after a period of eccentricity damping
- WORK IN PROGRESS:
  - Very late eccentricity growth in the massive case, prediction to be checked
  - Improve treatment of pumping and damping In the toy model
  - Eccentricity ratio between disc and planet in simulations do not fit well the toy model predictions