NATURAL SCIENCES TRIPOS Part II

Wednesday 28th May 2008 9.00 to 11.00

EXPERIMENTAL AND THEORETICAL PHYSICS (2)

Attempt the **whole** *of Section A, and* **two** *questions from Section B. Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to* **each** *question from Section B should be tied up separately, with the number of the question written clearly on the cover sheet.*

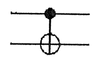
Section A carries approximately a quarter of the total marks. The **approximate** *number of marks allocated to each part of a question in Section B is indicated in the right margin. This paper contains 4 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
Script paper
Metric graph paper
Rough work paper
Blue coversheets
Tags

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculators allowed

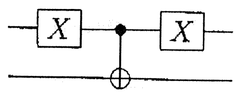
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

A4 In quantum computing, the gate X is represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The CNOT gate, depicted in a quantum circuit by



,

takes a 2-qubit input and flips the target qubit (the lower wire) if and only if the control qubit (the upper wire) is set to $|1\rangle$. Describe the effect of the circuit shown below.



SECTION B

B5 Write brief notes on **three** of the following: [22]

(a) the optical spectrum of sodium;

(b) the Stern–Gerlach experiment;

(c) Fermi's golden rule;

(d) quantum teleportation.

B6 Prove that, for a Hermitian Hamiltonian $\hat{H}$ with lowest eigenvalue E_0, any wavefunction ψ satisfies

$$\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0.$$

How is this result applied in the variational method for estimating eigenvalues? [5]

In some three-dimensional systems, the screened Coulomb potential

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}\exp(-\alpha r)$$

is a better approximation to the true potential than the Coulomb potential. A trial wavefunction

$$\psi(r) = \left(\frac{b^3}{\pi}\right)^{1/2} \exp\left(-br\right)$$

is to be used to estimate the ground state energy of an electron in such a system.

(TURN OVER

B7 Describe the method of degenerate perturbation theory. [5]

Explain the origin of the linear and quadratic Stark effects in the hydrogen atom. [3]

Taking $n = 2$ hydrogenic states of the form:

$$\begin{aligned}
\psi_{2s} &= \frac{1}{2\sqrt{2\pi}} a_0^{-3/2}(1 - r/2a_0)\exp(-r/2a_0), \\
\psi_{2p0} &= \frac{1}{4\sqrt{2\pi}} a_0^{-3/2}(r/a_0)\exp(-r/2a_0)\cos\theta, \\
\psi_{2p\pm1} &= \mp\frac{1}{8\sqrt{\pi}} a_0^{-3/2}(r/a_0)\exp(-r/2a_0)\sin\theta\exp(\pm i\phi),
\end{aligned}$$

show that the linear Stark effect due to an electric field $\mathcal{E}$ brings about a shift of energy of magnitude $3e\mathcal{E}a_0$ for two of these states, and give a normalised form for the wavefunctions of all four perturbed states. e is the electron charge, a_0 the Bohr radius. [7]

[*The integral* $\langle\psi_{2s}|r\cos\theta|\psi_{2p0}\rangle$ *is equal to* $-3a_0$.]

Sketch the energies of these four states, and of the 1s state, as a function of the electric field $\mathcal{E}$ for both positive and negative $\mathcal{E}$. Describe, with the help of sketches, how the mean position of the electron would vary if the atom is initially in the 2s state and the field is zero, *then* the field is gradually made positive, and *then* gradually reduced and made negative. [4]

Explain why only a small fraction of atomic states have a permanent electric dipole. [3]

END OF PAPER