1. Consider two point masses, each of mass $m$, separated vertically by a distance of 1 cm just above the surface of a neutron star of mass $M$ and radius $R$. Using Newton's Law of Gravity, derive an expression for the difference of the gravitational forces on the two masses. Evaluate the expression for $R = 10 \text{ km}$, $M = 1.4 \, M\odot$ and $m = 1 \, \text{kg}$.

Imagine a cube of iron, 1 cm on one side, falling towards the surface of a neutron star. The density of iron is 8000 kg m$^{-3}$. If iron experiences a stress (force per unit cross-sectional area) of $2 \times 10^9 \, \text{Nm}^{-2}$, it will rupture. Will the cube reach the surface of the neutron star intact?

2. The X-ray pulsar Centaurus X-3 has a period of 4.84 seconds and an X-ray luminosity of $5 \times 10^{30} \, \text{W}$. The pulsar is a neutron star with mass $M = 1.4 \, M\odot$ and radius $R = 10^4 \, \text{m}$. Estimate the accretion rate in units of $M\odot \, \text{yr}^{-1}$ for spherical accretion.

3. A pulsar that is not accreting material has a gamma-ray luminosity of $10^{31} \, \text{W}$ and a pulse period of $0.06 \, \text{s}$. Estimate a lower limit of $dP/dt$. If $P$ and $dP/dt$ have been measured, discuss how a limit on the distance can be obtained from the observed gamma-ray flux.

4. For a pulsar with a spin period of $P$, show that the spin-down luminosity is given by

$$-\dot{E} = A \frac{\dot{P}}{P^3}$$

and that the magnetic field strength is given by

$$B^2 = C P \dot{P}$$

where $A$ and $C$ are constants which you should determine (note: these differ for cgs and SI units). Furthermore, assuming that the pulsar’s period when it was born is short compared to its current measured period, show that the characteristic age of the pulsar, $\tau$, is given by

$$\tau = \frac{P}{2\dot{P}}$$

Sketch lines of constant spin-down luminosity, constant magnetic field strength and constant age in the $\log \dot{P} - \log P$ plane.
5. Let the observed duty cycle for pulsars be \( \eta \). Assume that all the neutron stars within some nearby volume have radio beams which are bright enough to be detected if they are aimed at the Earth. Estimate, in terms of \( \eta \), the fraction of these which can actually be seen and calculate this fraction for \( \eta = 5\% \).

6. Calculate the maximum power in gravitational waves that can be radiated from an object. How does this compare with the electro-magnetic radiation luminosity of the Milky Way galaxy?

7. Calculate the strain that has to be detected for a merging NS-NS binary at a distance of 15Mpc (assume both neutron stars have \( M = 1.4M_\odot \) and that the characteristic frequency is 100Hz). What would be the expansion/contraction of the test masses in LIGO? How does this compare to the size of an atom? What fraction of the LIGO laser wavelength (\( \lambda = 1\mu m \)) has to be measured?

8. As two massive bodies spiral in to each other the distance between them, \( a \), shrinks at a rate

\[
\frac{da}{dt} = -\frac{64 G^3 \mu M^2}{5 c^5 a^3}
\]

where \( M \) is the total mass and \( \mu \) is the reduced mass. Derive an expression for the time it takes for the two objects to coalesce and use your result to calculate how long it will take for PSR1913+16 (the Hulse-Taylor system) to coalesce. For PSR1913+16 use \( M_1 = M_2 = 1.4M_\odot \) and \( a_0 = 2 \) million km.

Ian Parry, Feb 2020.