**Topics in Observational Astrophysics**

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Lecture 7

- Neutron star – basics, sizes.
- Discovery of pulsars
- Pulsar spin speeds
- Basic model of a pulsar
- Pulse characteristics

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**Neutron Stars**

- What happens if the mass of a degenerate object exceeds the Chandrasekhar mass of approximately $1.4 \, M_\odot$?
- When the degeneracy energy of the individual particles is greater than the mass difference between a proton and an neutron we find that protons and electrons combine by inverse $\beta$ decay:
  \[
  e^- + p \rightarrow n + \nu
  \]
- This occurs for energies above:
  \[
  (m_n - m_p)c^2 \approx E_{deg} \approx 1.29\, MeV
  \]
- Just as with the electrons, neutrons are subject to degeneracy pressure. For a given mass, the radius of the star is inversely proportional to the mass of the dominant degenerate particle $m_{deg}$:
  \[
  R_{NS} \approx R_{WD} \left( \frac{m_e}{m_n} \right)
  \]
- As the mass of a neutron is approximately 2000 times that of an electron, the radius of a neutron star is approximately 2000 times smaller than that of a white dwarf held up by electron degeneracy pressure.
  \[
  m_n \sim 2000 \, m_e \quad M \propto \frac{1}{(m_{deg} R)^3}
  \]
  (We’ll make a more accurate estimate a few slides later).
- The radius of the sun, $R_\odot$, is approximately $7 \times 10^8$ metres. If we assume that the radius of a white dwarf is $2 \times 10^7$ metres we get a neutron star radius of $\sim 10^4$ metres or about 6 miles (about the size of Cambridge).
Neutron Stars

- Neutron stars are similar in some ways to white dwarfs in as much as they are supported against gravitational collapse by degeneracy pressure. For white dwarfs the degenerate particles are electrons whereas for neutron stars the degenerate particles are neutrons.
- Stars with $M > 8M_\odot$ will end their lives as a core-collapse supernova and create a neutron star or a black hole in the process.
- Neutron stars have remarkably extreme properties.
- You can work out that the neutrons are typically spaced by a distance of approximately $10^{-15}$ metres, a distance which is approximately the size of an atomic nucleus.
- This means that the star has a nuclear density of the order of $10^{14} \text{ kg m}^{-3}$.
- The gravitational potential energy of the star which is, of course, $GM^2/R$, is a substantial fraction of the rest mass energy, $\sim 0.1 M c^2$.
- This implies that general relativity is necessary when calculating accurately what happens on the surface of neutron stars and within their interiors.
- You can also work out that the mean velocity of the particles at the surface is $\sim 0.6 c$ and that the potential energy of a particle at the surface is approximately $0.2 M c^2$.
- The equation of state (EOS) for degenerate neutrons is not as well understood as the EOS for degenerate electrons.

Neutron Stars and Black Holes

- The mass-radius relation for neutron stars is similar to that of white dwarfs because pressure support comes from a degenerate gas. As the mass increases neutron stars also shrink.
- Eventually, as the mass increases, we reach a point where the degeneracy pressure provided by neutrons cannot support a neutron star.
- This leads to the collapse into a black hole, where the escape velocity of photons from the surface is greater than the speed of light and therefore no radiation can be emitted from the surface of the star.
- We believe (from observations) that for masses less than about 2.1 solar masses neutron stars can support themselves but beyond that they will collapse into a black hole.
- As with neutron stars we need general relativity to work out properly what is happening.
- Classically we would say that $v^2 = 2GM/R$. If we set $v = c$ then we get the Schwartzschild radius equal to the size of a black hole given as:
  $$R_{\text{schw}} = \frac{2GM}{c^2}$$
Relative Sizes of Stars: from Normal Stars to Black Holes

- $R_\odot \sim 7 \times 10^8$ metres
- $R_{\text{earth}} \sim 6 \times 10^6$ metres
- $R_{\text{WD}} \sim 5000$km
- $R_{\text{NS}} \sim 10$km

**Discovery of pulsars**

- The neutron was discovered in 1932. The existence of neutron stars was hypothesised in 1934 and linked to what might happen in a supernova (by Baade & Zwicky).
- This led to the prediction that there should be a magnetised neutron star in the centre of the Crab nebula in 1964 by Hoyle, Narlikar & Wheeler. The Crab nebula is a remnant of a supernova that exploded in 1054 AD, and was recorded by Chinese astronomers as a naked eye object for about 18 months.
- In 1967, a graduate student, Jocelyn Bell Burnell, in the Radio Astronomy group of the Cavendish laboratory in Cambridge was trying to track down the source of interference that she was seeing with her low frequency radio telescope. Her thesis project was to study the relatively rapid scintillations in radio intensity of distant radio sources (quasars) caused by the passage of the radio waves through the solar wind.
Discovery of Pulsars

- Jocelyn Bell Burnell was looking through the hours of pen-chart plots. In particular she was tracking down some interference which at the frequencies they were working at (81.5MHz or 3.7m) was rather serious. Today, BBC Radio 2 is at about 89MHz.
- She noted interference that occurred at a repeatable sidereal time (rather than solar time) indicating that its source was extra-terrestrial.
- The first object was called LGM1 (for little green men). The subsequent discovery of another three around the sky made it very unlikely that this was intelligent life.
- She and Tony Hewish (who later received a Nobel prize for this) had discovered a periodic extra-terrestrial signal of 1.337 s at position: RA 19:19:36, DEC +21:47:16
- This was the first Pulsar (PSR J1921+2153).
- Not seen previously because most radio astronomy requires long integrations on faint sources rather than the high time resolution they were using for scintillation studies of the solar wind.

Pulsars are Neutron stars

- The maximum rotation rate that a star can have is when the centripetal force at the surface is equal to the gravitational acceleration at that surface. Then $\omega^2 R = GM/R^2$, and the minimum rotation period is $P_{\text{min}} = 2\pi/\omega = 2\pi(R^3/GM)^{1/2}$.
- This gives a minimum period of $P_{\text{min}} \sim 7$ sec for a white dwarf but since $R_{\text{wd}}/R_{\text{ns}} \sim 500$, then $P_{\text{min}} \sim 5 \times 10^{-4}$ sec (2kHz), which can be compared with one of the faster pulsars, the Crab pulsar, which has a period of $3.33 \times 10^{-2}$ sec or 30Hz. The fastest one known has $P \sim 716$ kHz.
- What do we expect for the actual rotation period of a neutron star? The precursor of a neutron star is a stellar core made out of $^{56}$Fe. The mass and radius of a white dwarf (which is similar to the stellar core) are related by:

$$R_{\text{wd}} \propto \frac{1}{m_w M_{\text{wd}}^{1/3}} \left( \frac{Z}{A} \frac{1}{m_n} \right)^{5/3}$$

- And for a neutron star this is given by:

$$R_{\text{ns}} \propto \frac{1}{M_{\text{ns}}^{1/3}} \left( \frac{1}{m_n} \right)^{8/3}$$

- $Z = \text{nuclear charge}, A = \text{nuclear mass}$ so:

$$\frac{Z}{A} \approx \frac{26}{56} \approx 0.464 \text{ for Fe}$$

- And given that $M_{\text{wd}} \sim M_{\text{ns}}$ we see that:

$$\frac{R_{\text{wd}}}{R_{\text{ns}}} \sim \frac{m_n}{m_e} \left( \frac{Z}{A} \right)^{5/3} \sim 500$$

- Showing that the radii are very different: 
Pulsar Properties

• Here we assume the angular momentum of a white dwarf and the angular momentum of a neutron star are the same. This then gives us an estimate of the pulsation period.

• The moment of inertia of a sphere is given by \( I = cMR^2 \), with a constant \( c \) which depends on the density as a function of radius. For example, for a uniform density we have \( c = 2/5 \).

\[
I = cMR^2
\]

• Equating angular momenta gives us:

\[
l_{\text{WD}}\omega_{\text{WD}} = l_{\text{NS}}\omega_{\text{NS}}
\]

\[
M_{\text{WD}}R_{\text{WD}}^2\omega_{\text{WD}} = M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}}
\]

• Also, assume \( M_{\text{WD}} \sim M_{\text{NS}} \)

\[
\Rightarrow \omega_{\text{NS}} = \omega_{\text{WD}} \left( \frac{R_{\text{WD}}}{R_{\text{NS}}} \right)^2
\]

or

\[
P_{\text{NS}} = P_{\text{WD}} \left( \frac{R_{\text{NS}}}{R_{\text{WD}}} \right)^2 \Rightarrow P_{\text{NS}} \sim 5 \times 10^{-3} \text{ sec}
\]

• White dwarfs are observed to have periods of about 1500 seconds and had been the fastest rotators until pulsars were found.

• We saw earlier that pulsars are about 500 times smaller than white dwarfs and this is why they rotate so rapidly.

• To put this in context, the sun is observed to rotate in about 25 days and a white dwarf in 1000-10,000 seconds (they are 100 times smaller than the sun typically).

Pulsar Properties

• So neutron stars are capable of explaining pulsar periods. Indeed, the periods we observe are not surprising given what we already know about normal stars, white dwarfs, and angular momentum.

• We still need explanations for:

1. The very wide range of rotation periods (these are observed to range from a few seconds down to about one millisecond).
2. The fact that the pulses are only on for a very small fraction of the total period. Typically the duty cycle is about 5%.
3. The pulses are detectable over a very large range of frequency. The Crab pulsar is seen at radio, optical, x-ray and gamma-ray wavelengths. (Most pulsars are seen at radio wavelengths only).

• The inter-pulse emission implies that they are radiating as a black body with a characteristic temperature and wavelength but the pulse emission mechanism must be of a very different, non-thermal origin.

• Let’s now look at what we think a pulsar is and look at the wide range of observational data that supports this view.
Radio beam is due to curvature radiation – similar to synchrotron radiation but the charged particle is moving along the field line and is accelerated because the field line is curved.

Light cylinder is where the solid-body rotation velocity is the speed of light.

Beam shape can have two cones. Pulse structure depends on viewing angle. Cone angles depend on wavelength.
Pulsar Properties

- The picture that we have is of a neutron star with a strong embedded magnetic field set at an angle to its rotation axis.
- The highly charged relativistic plasma near the surface can only be ejected parallel to the magnetic field.
- This produces a substantial beaming of radiation in the direction of the magnetic axis so that the distant observer only detects a pulse when the magnetic axis is pointing close to the line of sight.
- This also makes it clear that only a small proportion of pulsars will be visible. If the pulse duty cycle is only 5%, then roughly only 5% of pulsars will be visible.


Pulsar Properties: where are they found?

- Early searches for a pulsating optical counterpart of the original pulsars were not successful. The first optical pulsations that were detected were from the Crab nebula.
- Only six pulsars have been detected now at optical wavelengths but a great many are detected in the vicinity of supernova remnants.

The individual radio pulses from pulsars are quite different from one another. They vary in shape and amplitude indicating a fairly complex and changing geometry for the region that generates the radio pulses that we see.

The pulse shape obtained from adding together a few hundred pulses is quite stable and repeatable. Pulse drift implies cones are wandering but in a repeatable sort of way?
The pulse structure varies in quite a complicated way as a function of wavelength. The above picture shows a single pulse that has been detected by different radio telescopes around the world which were all synchronised with great accuracy. There is a tendency for the low frequency pulses to be broader and more separated than they are at high frequencies.

Two more examples of pulse shape versus observed frequency showing that the width is narrower for higher radio frequencies.
• This suggests the sort of model where the highest frequencies are emitted closer to the pulsar and that the broadening may be caused by the divergence in the magnetic field lines.