Lecture 17

• Primary CMB fluctuations
  – The seed density perturbations
  – Sachs-Wolfe effect
  – Peculiar velocities
  – Acoustic peaks
  – Silk damping

• CMB observations v theory

• CMB combined with SNIa results
The primordial density fluctuation seeds

Q: Where do the inhomogeneities on the last scattering surface come from?

A: The seeds which produce the density perturbations on the last scattering surface are created in the reheating phase during inflation which produces small inhomogeneities due to quantum fluctuations.

This happens in a way that the initial fluctuations of the gravitational potential are essentially independent of scale. However, a small departure from this, in the form of a spectral index, is now part of the best fitting model. See

http://map.gsfc.nasa.gov/resources/camb_tool/cmb_plot.swf
Consider a sphere of (radius $\tilde{R}$) in material field with mean density $<\rho>$.

Define the mass variance

\[ \sigma_m^2 \equiv \frac{\langle (M - \langle M \rangle)^2 \rangle}{\langle M \rangle^2} = \frac{\langle \Delta M^2 \rangle}{\langle M \rangle^2} \]

It can be shown that

\[ \sigma_m \propto M^{-2/3} \]

Due to the sphere the gravitational potential is changed by

\[ \Delta \Phi \propto \frac{G \Delta M}{\tilde{R}} \propto \frac{G \Delta \rho}{\tilde{R}} \frac{\tilde{R}^3}{\tilde{R}} \propto G\langle \rho \rangle \sigma_m \tilde{R}^2 \propto G\langle \rho \rangle \sigma_m \tilde{R}^2 \propto \sigma_m M^{2/3} = \text{constant} \]

\[ \Delta \Phi = \text{const, i.e. independent of } \tilde{R} \]

This is called a Harrison-Zeldovich or a scale-free fluctuation spectrum.
Primary CMB fluctuations

Gravitational/redshift time dilation

If a photon climbs out of a potential “well” or falls down a potential “hill” it is red-shifted or blue-shifted.

\[
\frac{\Delta v}{v} = \frac{\Delta T}{T} = \frac{\Delta \Phi}{c^2} \quad (=\text{const. for Harrison-Zeldovich fluctuation spectrum})
\]
There is also a time dilation effect

\[ \frac{\Delta t}{t} = \frac{\Delta \Phi}{c^2} \]

\[ T = \left( \frac{t}{t_o} \right)^{-2/3} \] in matter dominated phase

\[ \frac{dT}{T} = -\frac{2}{3} \left( \frac{t}{t_o} \right)^{-5/3} \frac{dt}{t_0} \left( \frac{t}{t_o} \right)^{2/3} = -\frac{2}{3} \left( \frac{t}{t_o} \right)^{-1} \frac{dt}{t_0} = -\frac{2}{3} \frac{dt}{t} \]

\[ \frac{\Delta T}{T} = \frac{\Delta \Phi}{c^2} - \frac{2}{3} \frac{\Delta \Phi}{c^2} = \frac{1}{3} \frac{\Delta \Phi}{c^2} \] Sachs-Wolfe effect

gravitational redshift  time delay  Scale invariant
Due to fluctuations in the potential, the matter on the last scattering surface moves with respect to the Hubble flow with a peculiar velocity $v_{\text{pec}}$

$$v_{\text{pec}} \sim g \ t_H \left( z = z_{\text{rec}} \right) \sim g \ t_{\text{rec}}$$

acceleration

age of Universe at recombination
Estimate the acceleration

\[ g \sim \frac{G \Delta M}{\tilde{R}^2} \sim \Delta \Phi \frac{1}{\tilde{R}} \]

\[ \frac{\Delta T}{T} \sim \frac{v_{\text{pec}}}{c} \sim \Delta \Phi \frac{c}{c^2} \frac{t_{\text{rec}}}{\tilde{R}} \]

\( c \ t_{\text{rec}} \) is about the horizon size at recombination (see lecture 12)

\[ r_{\text{hor}} (t_{\text{rec}}) = c \int_0^{t_{\text{rec}}} \frac{dt}{R(t)} = \int_0^{t_{\text{rec}}} \frac{dt}{R_0 \left( \frac{t}{t_0} \right)^{2/3}} = \frac{3}{2} \frac{c}{R_0} \left( \frac{t_{\text{rec}}}{t_0} \right)^{1/3} \]

\[ R (t_{\text{rec}}) \ r_{\text{hor}} (t_{\text{rec}}) = 3c \ t_{\text{rec}} = \tilde{R}_{\text{hor}} (t_{\text{rec}}) \]
Recall $\Theta_{\text{hor}}(t_{\text{rec}}) \sim 2\text{deg}$ in an Einstein-de Sitter Universe. Doppler effect dominates over the S-W effect on scales smaller than about 2deg.

$$\left( \frac{\Delta T}{T} \right)_{\text{doppler}} = \frac{1}{3} \frac{\Delta \Phi}{c^2} \frac{\tilde{R}_{\text{hor}}(t_{\text{rec}})}{\tilde{R}}$$

$$\left( \frac{\Delta T}{T} \right)_{\text{grav}} = \frac{1}{3} \frac{\Delta \Phi}{c^2}$$
The Acoustic Peaks

- Basically they are due to standing wave oscillations within the sound horizon.
- The first peak used to be called the Doppler peak because it is where Doppler effects dominate.
- The CMB is a snapshot of the acoustic motions of the gas when the pressure drops to zero.
- The position of the first peak is strongly related to $\Omega_k$. This is because the physical length which makes the peak is essentially the sound horizon size $c_s t_{rec}$ which only has a weak dependence on cosmological parameters.
- However, the observed angular size depends strongly on $\Omega_k$ (think about the angle between two geodesics).
Diffusive damping

Photon diffusion will damp fluctuations on small scales. How far do the photons get at recombination?

The time between scatterings is

\[ t_{\text{scat}} = \frac{1}{n_e \sigma_T c} \]

The number of scatterings is

\[ N_{\text{scat}} = \frac{t_{\text{rec}}}{t_{\text{scat}}} \]

\[ t_{\text{rec}} = \int_{\infty}^{z_{\text{rec}}} - \frac{dz}{(1+z)H(z)} \approx \int_{\infty}^{z_{\text{rec}}} - \frac{dz}{\Omega_{\text{mat},0}^{1/2} H_0 (1+z)^{5/2}} \]

\[ = \frac{2}{3} \frac{1}{H_0} \frac{1}{\Omega_{\text{mat},0}^{1/2}} (1+z_{\text{rec}})^{-3/2} \]
Mean free path \[ l_{\text{scat}} = \frac{1}{n_e \sigma_T} \]

Diffusion length \[ L_{\text{diff}} = (N_{\text{scat}})^{1/2} l_{\text{scat}} \]

\[
L_{\text{diff}} = \left( \frac{2}{3} \frac{1}{H_0} \Omega_{\text{mat,0}}^{1/2} \left(1 + z_{\text{rec}}\right)^{-3/2} \right) \left( n_e \sigma_T c \right)^{1/2} \frac{1}{n_e \sigma_T}
\]

\[
= \left( \frac{2}{3} \frac{1}{H_0} \Omega_{\text{mat,0}}^{1/2} \left(1 + z_{\text{rec}}\right)^{-3/2} \right) \frac{c}{n_e \sigma_T}^{1/2}
\]

\[ n_e = \rho_{\text{crit,0}} \chi_e (1 + z)^3 \Omega_{\text{bar}} = \frac{3H_0^2}{8\pi G} \chi_e (1 + z)^3 \Omega_{\text{bar}} \]

Physical Cosmology 2011/2012
Plugging in the values for $\chi_e$, $c$, $G$ and $z_{\text{rec}}$ we get

$$\text{At } z = z_{\text{rec}} \quad L_{\text{diff}} = 2.7 \left( \Omega_{\text{mat},0} \Omega_{\text{bar}}^2 h^6 \right)^{1/4} \text{ Mpc}$$

This is called the Silk length. Fluctuations on smaller scales are washed out.

Corresponding angular scale is $\Theta_{\text{diff}} \sim 1.5$ arcmin

On scales $\Theta_{\text{diff}} < \Theta < \Theta_{\text{hor}}(t_{\text{rec}})$ the coupled baryon-photon fluid on the last scattering surface oscillates.

These oscillations are phase-coherent for fluctuations on a given scale.
These primary fluctuations result in a characteristic pattern of the temperature fluctuations as a function of scale.
The Boomerang experiment

http://cmb.PHYS.cwru.edu/boomerang
The Boomerang flight path
Ground-based data

The acoustic oscillations have been detected by a variety of ground-based/balloon instruments.

http://www.hep.upenn.edu/~max/
The angular power spectrum as measured by WMAP

\[ \left< \left( \frac{\Delta T}{T} \right)^2 \right> \]

WMAP results have small error bars

\[ l(l+1)C_l/2\pi (\mu K^2) \]

TT Cross Power Spectrum

- $\Lambda$ - CDM All Data
- WMAP
- CBI
- ACBAR

http://map.gsfc.gov/
The location of the first peak and $\Omega_{k,0}$

The angular scale of the first peak scales with

$$\Theta_{\text{hor}}(t_{\text{rec}}) = \frac{R(t_{\text{rec}}) r_{\text{hor}}(t_{\text{rec}})}{D_{\theta}(z_{\text{rec}})} = \frac{3 c t_{\text{rec}}}{D_{\theta}(z_{\text{rec}})}$$

$$t_{\text{rec}} \approx \frac{2}{3} \frac{1}{H_0} \frac{1}{\Omega_{\text{mat},0}^{1/2}} (1 + z_{\text{rec}})^{-3/2} \propto \Omega_{\text{mat},0}^{-1/2}$$

$$D_{\theta}(z_{\text{rec}}) = \frac{1}{1 + z_{\text{rec}}} \frac{c}{H_0} \int_0^{z_{\text{rec}}} \frac{dz}{\sqrt{\Omega_{\text{mat},0} (1 + z)^3 + \left(1 - \Omega_{\text{mat},0} - \Omega_{\Lambda,0}\right) (1 + z)^2 + \Omega_{\Lambda,0}}}$$
The location of the peaks shift to smaller angular scale for increasing $\Omega_k,0 = 1 - \Omega_{\text{mat},0} - \Omega_{\Lambda,0}$ with fixed $\Omega_{\text{mat},0} h^2$ and $\Omega_{\Lambda,0}$.
The height of the peaks increases/decreases with increasing/decreasing baryon/dark matter density.

Increasing dark matter density leads to increased damping of the oscillations, while increased baryon density results in an increased pressure and larger amplitude.
The angular power spectrum also depends on $h$ and $\Omega_{\Lambda,0}$.

The analysis is complicated and the dependence on the parameters is degenerate.

Need to combine with other data to constrain $\Omega_{\Lambda,0}$, $H_0$, $\Omega_{\text{bar}}$.
Recall the Supernova result

Likelihood contours in $\Omega_{\text{mat},0} - \Omega_{\Lambda,0}$ plane

$$q_0 = \frac{\Omega_{\text{mat},0}}{2} - \Omega_{\Lambda,0} = 0$$

http://www-supernova.lbl.gov
Combining supernovae and CMB results.

Combination of data sets breaks degeneracies.

\[ \Omega_{\text{mat},0} \approx 0.3 \]

\[ \Omega_{\Lambda,0} \approx 0.7 \]
Main results

Universe is close to or exactly flat \((\Omega_k,0 \sim 0)\)

Initial fluctuation spectrum is very close or identical to an Harrison-Zeldovich spectrum \(\leftrightarrow\) Inflation

By combining with other data sets the relative fraction of dark matter/ dark energy and baryons are determined.
The concordance model of cosmology

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\Omega_{\text{tot}}$</td>
<td>$1.02^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>$w$</td>
<td>$&lt;-0.78$ (95% CL)</td>
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<tr>
<td>$\Omega_0$</td>
<td>$0.73^{+0.04}_{-0.04}$</td>
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<tr>
<td>$\Omega^h_0h^2$</td>
<td>$0.0224^{+0.0009}_{-0.0009}$</td>
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<tr>
<td>$\Omega_b$</td>
<td>$0.044^{+0.004}_{-0.004}$</td>
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<tr>
<td>$n_b$</td>
<td>$2.5 \times 10^{-7}^{+0.1 \times 10^{-7}}_{-0.1 \times 10^{-7}}$ cm$^{-3}$</td>
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<tr>
<td>$\Omega_m$</td>
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<td>$\Omega^h_mh^2$</td>
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<tr>
<td>$\Omega^v_mh^2$</td>
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<td>$m_v$</td>
<td>$&lt; 0.23$ eV (95% CL)</td>
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<tr>
<td>$T_{\text{cmb}}$</td>
<td>$2.725^{+0.002}_{-0.002}$ K</td>
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<tr>
<td>$n_\gamma$</td>
<td>$410.4^{+0.9}_{-0.9}$ cm$^{-3}$</td>
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<tr>
<td>$\eta$</td>
<td>$6.1 \times 10^{-10}^{+0.3 \times 10^{-10}}_{-0.2 \times 10^{-10}}$</td>
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<tr>
<td>$\Omega_0\Omega^h_0^{-1}$</td>
<td>$0.17^{+0.01}_{-0.01}$</td>
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<tr>
<td>$\sigma_8$</td>
<td>$0.84^{+0.04}_{-0.04}$ Mpc</td>
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<td>$\sigma_8\Omega^{0.5}_m$</td>
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<td>$A_s$</td>
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<td>$n_s$</td>
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<td>$dn_s/d\ln k$</td>
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<tr>
<td>$r$</td>
<td>$&lt; 0.71$ (95% CL)</td>
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<td>$\Delta z_{\text{dec}}$</td>
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<td>$t_0$</td>
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