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VARIATIONS OF ATMOSPHERIC TURBULENCE

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In this paper we examine spatial and temporal variations of the parameters of atmospheric turbulence--with particular emphasis on the region above the earth's boundary layer.

The variations of path statistics are of obvious importance to system designers who are concerned with extreme value statistics. Communications engineers worry about worse case statistics for error rate analyses, while planetary astronomers are happy to select the best of many images. Additionally, any researcher attempting to critically test a theory (such as lognormality of intensity fluctuations) desires that the path statistics be constant during the test. This is often not the case. The conventional rules for accuracies of averages versus averaging time may not apply.[1]

The variations of turbulence statistics come in many size scales. The smallest of these (on a scale of tens of centimeters) is a phenomenon known as intermittency. An example is given in Figures 1, 2 and 3 of Reference 2. It has been proposed by Obukhov, Kolmogoroff, Yaglom and others that various turbulence strength parameters are lognormally distributed. Chen[3] and others have shown that the strength distributions cannot be exactly lognormal, but there is ample experimental evidence that over volume averages lognormality is a reasonable approximation. Reference 4 contains a detailed review of the theory of intermittency. Intermittency will very slightly modify the slope of the classical $k^{-5/3}$ spectral law. More significant, however, is the prediction that local gradients of the index of refraction are not normally distributed, an effect which may yield excess kurtosis in small aperture angle of arrival statistics over short paths. The lognormality of scintillation over short paths may also be affected.

Near the earth's boundary layer one observes turbulent thermal plumes, in active convective conditions. This has been discussed by Webb and Coulman, and has a scale of hundreds of meters[5,6].

Variations in wind speed will convert even a spatially invariant spectrum into a temporally varying one. This effect is most obvious over short horizontal paths, but is present in more subtile forms over any path. As the sun changes elevation angle, or is obscured by clouds, the ground heating changes and with it the turbulence. Again the effect is straightforward.

Observations of turbulence in the upper boundary layer, $30m \leq h \leq 3km$, have been made by direct in situ observation[2,7,8] and by remote sensing. From the direct measurements by Ochs and Lawrence it is apparent that there is much detailed dynamic fine structure to the $C_N(h)$ profile, but an insight into the physics of the process requires continuous monitoring of the atmosphere. The acoustic sounders, pioneered at NOAA and in Australia provide just what is needed. The results are fascinating to look at[9,10], and a much better understanding of the physics of the atmosphere in this region cannot be far ahead. What is already clear is that atmospheric gravity waves play a vital part of the total process.

With such intensive investigations currently underway by many workers, we can hope that improved physical models for the spatial and temporal variations of turbulence in the earth's boundary layer will soon follow.

Above a few kilometers the density of knowledge decreases. Direct in situ measurements of temperature fluctuations have been made by Bufton, et al using balloon borne probes[11,18]. Under the assumption that the observed fluctuations are entirely due to Kolmogoroff type turbulence, $C_N(h)$ profiles may be inferred. Remote probing by radar techniques has revealed the presence of turbulence layers, often associated with unstable gravity waves[12]. Stellar scintillation measurements reveal some information about the $C_N(h)$ profile, especially if joint spatial and temporal filtering techniques are used[13]. There are no neat universal quantitative theories for turbulence in this region.

The remainder of this paper will be devoted to the construction of a heuristic model for a dynamic $C_N(h,t)$ profile in this region. We will do this by assembling the available experimental evidence and then by synthesizing a physically reasonable mathematical process which agrees with this evidence. First the evidence:

(A) Examination of the measured $C_N(h)$ profiles by Bufton, et al [11], reveals large fluctuations about the median value. It appears both here and in the NOAA data[2,7] that C_N^2 behaves approximately as a lognormally distributed random variable. Log $C_N(h)$ becomes partially decorrelated over h differences of only $\sim$100 meters, but some correlation exists for Δh as large as 2km. There is some slight evidence for larger C_N values at the base of strong inversion layers. But in the absence of more data, there appear to be no overwhelmingly large peaks that could not be viewed as a normal statistical variation about a mean profile. However, the winds were always low during these experiments.

(B) Vinnichenko and Dutton[14] report on turbulent temperature fluctuations in the lower stratosphere over scales of $\sim$30m and longer. From their data one can derive the correlation $C_T \simeq 3\times10^{-3}\langle\varepsilon\rangle^{1/3}$ (cgs units) accurate to ±50%, where $\langle\varepsilon\rangle$ is the viscous dissipation rate.

(C) Hardy[12], using high power radar facilities, reports that during a period of intensive measurements "every clear-air radar echo above 3km was associated with aircraft reports of at least some perceptible degree of turbulence". Furthermore he states that there is "substantial evidence that strong CAT is associated with strongly stable zones where the vertical wind shear has developed considerably". Radar echos and CAT zones have typical horizontal dimensions of 5 to 15km. The probability of encountering significant CAT is only a few percent. From (B) above, significant CAT would correspond to lower stratospheric values of C_N^2 greater than $3\times10^{-17}m^{-2/3}$.

(D) Numerous quantitative observations of stellar scintillation have been made by Mikesell[15], Demidova[16], and Bufton[17]. We have done extensive statistical analyses of their data, and have coupled some of Mikesell's data with the original weather bureau radiosonde data for the times of observation. We have used only near zenith observations, and have reduced all scintillation data to correspond to zenith values with a 10cm aperture. The reduced value of $\sigma^2/\langle p\rangle^2$ we will call S for the remainder of the paper. S is proportional to a weighted path integral of $C_N^2(h)$ where the weighting factor increases somewhat less rapidly than h^2.

(1) Over a year long period log S is approximately a Gaussian random variable with σ=log 2.1.

(2) Two values of log S have ∿.75 correlation over a 40 minute time separation, ∿.6 correlation over 3 hours, ∿.5 over 24 hour separations, ∿.25 correlation over 48 hours, and very little correlation for longer separations. (These numbers do not have high statistical reliability.) Figure 1 shows an exceptionally dynamic event from Reference 15.

(3) The year round median value of S is about 0.06 for all sites.

(4) We attempted to correlate the scintillation spectrum and the value of S with various meteorological parameters, such as peak wind speed, speed at the tropopause, speed at significant inversions, and speed at regions of low Richardson's number. The best correlating factor was W^2. We define

$$W = \left[(1/15\text{km})\int_{5\text{km}}^{20\text{km}} V^2(h)\,dh\right]^{1/2} \qquad \text{(units of m/sec)}$$

where V(h) is wind speed at altitude h. About 60% of the total variance of log S is accounted for by this factor.

Based on the preceding empirical observation we have synthesized the following model for $C_N(h,t)$ valid for (ground+3000m)<h<24000m.

$$C_N^2 = \left\{\left[(2.2\times10^{-53})h^{10}(W/27)^2\right]\exp(-h/1000) + (10^{-16})\exp(-h/1500)\right\}\exp\left[r(h,t)\right] \qquad (\text{in units of } m^{-2/3})$$

where h is in meters above sea level and r is a zero mean homogeneous Gaussian random variable with a covariance function given by $\langle r(h+h',t+\tau)r(h,t)\rangle = A(h'/100)e^{-\tau/5} + A(h'/2000)e^{-\tau/80}$. The time interval τ is measured in minutes and $A(h'/L)=1-|h'/L|$ for $|h'|<L$ and zero otherwise. It follows that $\langle r^2\rangle=2$ and that $\langle\exp r\rangle=e=2.7$, a result which may be used for those not interested

in the fine structure. Further time variations are implicit in the statistics of W. For Maryland W appears to be normally distributed with an average value of 27m/sec and a standard deviation of 9m/sec. Conceivably the coefficients in the model should be different over mountainous terrain. This model is simple in many ways. It does not explicitly account for possible high turbulence peaks generated at frontal interfaces or sharp inversion layers because we had no statistics for these. Nevertheless, it does seem to yield reasonable agreement with most of the observed evidence. Figure 2 shows a typical sample $C_N^2(h)$ profile. To make the model complete for low altitudes, one may add a boundary layer term which depends on the altitude above the local ground level.

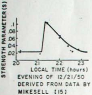

FIG. I. TURBULENCE STRENGTH VS. TIME

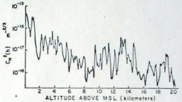

FIG. 2. SAMPLE OF RANDOM MODEL ATMOSPHERE (W=18 m/s)

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