

Collisional dust production in debris discs

MAIN ISSUES

- Size distribution in collisional cascades producing the dust observed in debris discs
- Derive the *unseen* population of parent bodies up to Km-sized objects
- Derive observational constraints: S.E.D, luminosity profiles

I context

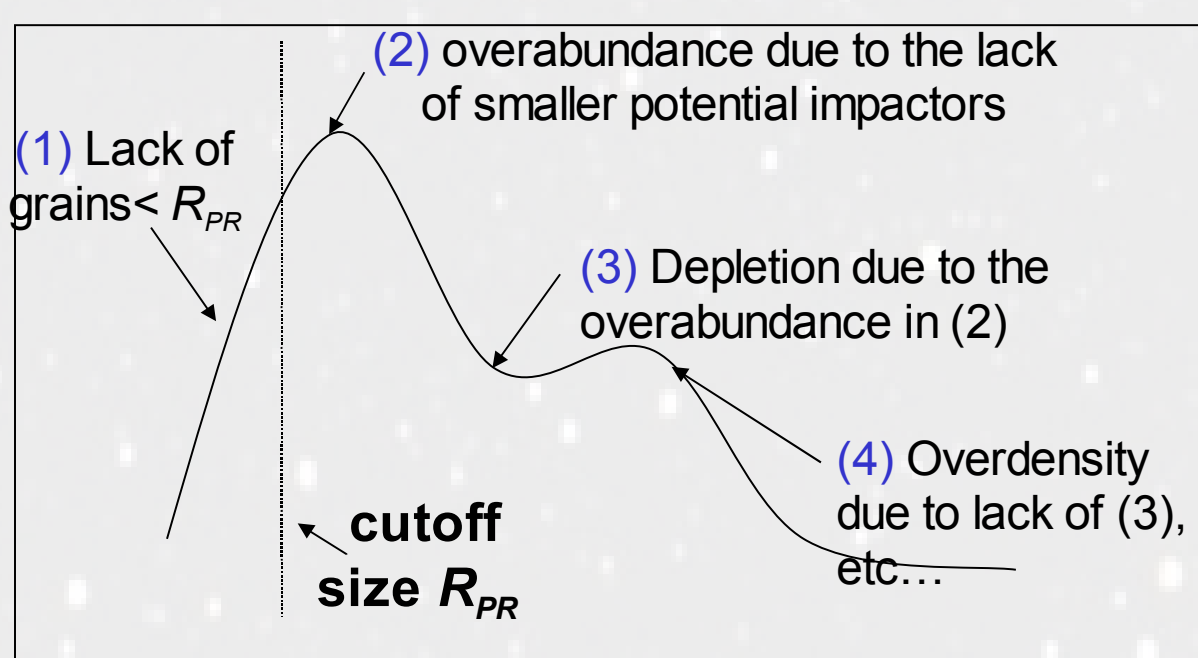
INTRODUCTION

Dust is usually observed in the μm to mm range. In many discs, simple estimations show that this dust cannot be primordial and has to be produced by collisional cascades from *unseen parent bodies*

The « usual » way to derive the parent bodies population is to assume a collisional equilibrium power law $dn \propto n^{-3.5} dr$ (Dohnanyi, 1969) and to extrapolate it up to the km-sized range (e.g. Artymowicz, 1997 for β -Pictoris, Augereau et al., 1999, for HR7496).

BUT...Problems with the $dn \propto n^{-3.5} dr$ law:

- very small differences in the power law index q can lead to major differences when extrapolated over 7 or 8 orders of magnitudes.
- the $n^{-3.5}$ power law supposes that the whole system has reached a collisional equilibrium, which is far from being certain in some young discs
- the $n^{-3.5}$ power law **applies only to theoretical systems with no minimum size cutoff**. Whereas here, **radiation pressure blows out all grains smaller than a cutoff size R_{PR}** . This might induce strong discrepancies with the Dohnanyi equilibrium (« wavy size distributions », see Campo Bagatin et al., 1994)



- Small grains have a peculiar dynamical behaviour. Particles close to R_{PR} are placed on high eccentricity orbits by radiation pressure
- higher impacting velocities Δv and thus shattering power
- wide spacial spreading of the smallest grains, which might collisionally affect areas far from their production region

NEW MULTI-ANNULUS NUMERICAL MODEL

In Thébault et al.(2003) (TAB03) we quantitatively studied these effects for the specific case of the inner (inside 10AU) β -Pic disc. For this purpose, a statistical numerical code was developed, which quantitatively follows the size distribution evolution of a population of solid bodies, in a wide μm to km size-range, taking into account the major effects induced by radiation pressure on the smaller grains (size cutoff, perturbed dynamical behaviour,...). Our main result was to identify an important departure from the $dn \propto n^{-3.5} dr$ law, especially in the 10^{-4} to 1cm range. The main limitation of this code was that it was single annulus, i.e. that it could only study a limited region at one given distance from the star ($\approx 5\text{AU}$) but not the system as a whole. To do so, a **multi-annulus approach** is needed. Such multi-annulus codes have been recently developed by Kenyon&Bromley (2004,2005) who have applied them to various different contexts. These codes are in some respect more sophisticated than the one used in TAB03, in particular because they follow the dynamical evolution of the system (which is fixed in TAB03). Nevertheless, the price to pay for following the dynamics is that the modelisation of the small grain population is very simplified, with all bodies $< 1\text{m}$ following an imposed $R^{-2} dR$ power law, thus implicitly overlooking the aforementioned consequences of the specific behaviour of the smallest dust particles. More recently Krivov et al.(2006), using a different approach, developed a model able to follow the evolution of both physical size and spatial distribution (1D) of a collisionally evolving idealized debris disc. This innovative approach gave promising result for the peculiar case of the Vega system. However, the modelisation of collisional outcomes is, as acknowledged by the authors themselves, very simplified, with for instance all cratering impacts being neglected. The present work is much in the spirit of Krivov et al., although we chose to keep the classical particle-in-a-box approach and the advanced modeling of collision outcomes of TAB03. We have developed a multi-annulus version of our earlier code in order to extend the approach of TAB03 to the general study of complete debris disc.

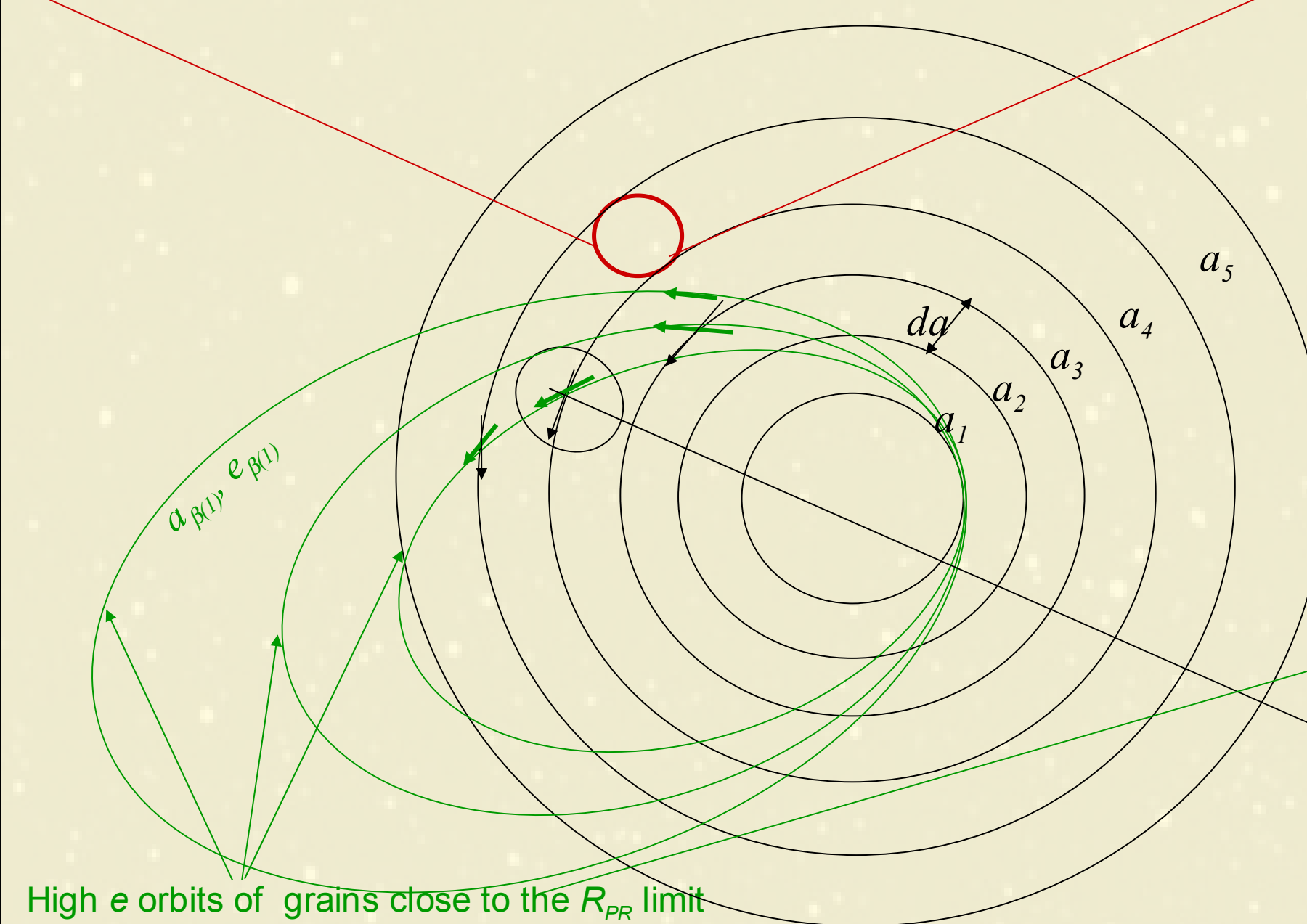
II numerical model

$$p_{i,j} = \frac{N_j}{4\pi a_i \Delta a_i} \langle \Delta v_{ij} \rangle (R_i + R_j)^2 dt$$

- « classical » particle-in-a-box statistical model in each annulus $[a - da/2, a + da/2]$
- n boxes spaced by a factor 2 in mass (i.e. 1.26 in size) between R_{PR} (cutoff size) and $R_{max} = 50\text{km}$.
- Evolution equation

$$dN_k = \sum_{i=1}^n \sum_{j=1}^n f_{i,j,k} p_{i,j} N_i N_j dt$$

$$\langle \Delta v \rangle = (5/4 \langle e^2 \rangle + \langle i^2 \rangle)$$
- Threshold specific energy Q_c : Benz & Asphaug (1999) (nominal case)
- Fragmentation-produced fragments power law:
- Craterization-produced fragments power law: Petit & Farinella, 1993



Collision outcome prescription

Shattering Energy Q^*

Nominal case:
 $Q_{sh}^* = 3.5 \times 10^7 (R/1\text{cm})^{0.28} + 0.3p(R/1\text{cm})^{1.38}$ (Benz&Asphaug, 1999)
 $Q_{ice}^* = 1/5 Q_{sh}^*$ (Krivov et al., 2005)
 Other explored prescriptions: Housen&Holsapple + Davis et al.

Fragmentation

- Largest fragment $M_f = 0.5(Q^* M_i / E_{col})^{1/24}$ (Fujiwara, 1977)
- Fragment size distribution: 2 index power law q_1 and q_2 (to avoid supercatastrophic impacts, see TAB03)

Cratering

- small scale craters $M_{cr} = K_c p E_{col}^{1/24}$ ($K_{ice} = 4.2 \times 10^{-4}$, $K_c = 3.10^{-7}$) ($E_{col} < 0.01 Q^*$) (Koschny&Grün, 2001)
- intermediate case: interpolation
- large scale craters $M_{cr} = 0.5 M_i (E_{col}/Q^*)$ (Wyatt&Dent, 2002) ($0.2 Q^* < E_{col} < Q^*$)

Specific behaviour of the smallest grains

Grains with high β are placed on high e and a orbits by radiation pressure, following:
 $a_f = a_i \left(\frac{1-\beta}{1-2\beta} \right)$, $e_f = \left(\frac{1-2\beta\beta_f/|1-\beta|}{1-\beta} \right)^{1/2}$

grains produced in a given initial annulus collisionally interact with the whole population of several external annuli

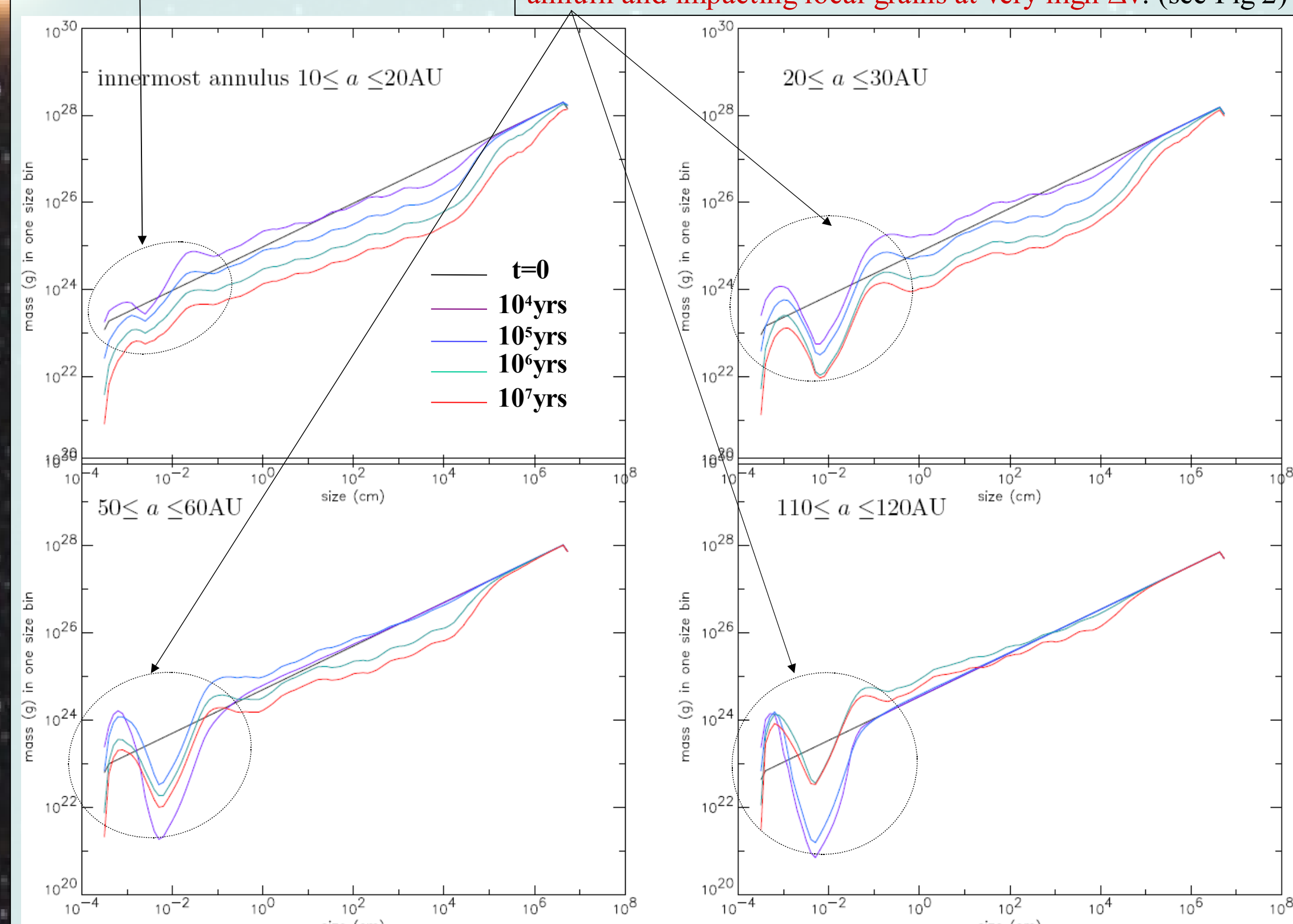
necessity for a numerical estimation of: (using a separate deterministic integrator)

- $f_{i(m)}$, fraction of bodies of size R_i produced in annulus a_i that reach annulus a_m
- $dt_{i(m)}$, average time spend, in the a_m annulus, by a R_i body produced in annulus a_i
- $\langle \Delta v_{(i,m)} \rangle$, average impact velocity, within the a_m annulus, between a R_i particle initially produced in the a_i annulus and a locally produced R_j body.

III simulations

Innermost annulus: wavy-pattern, but weaker than in TAB03, because: -lower $\langle \Delta v \rangle$ (15AU instead of 5AU) -more realistic cratering prescription

More pronounced wavy-patterns in the outer annuli! Counter-intuitive, because: lower Δv and longer dynamical timescales. BUT: -Most impacts are due to high β grains coming from the inner annuli and impacting local grains at very high Δv ! (see Fig 2)



Most important Features:

- **Steady state** sets in in $\sim 10^6$ yrs, with:
- **Overabundance** of bodies with $R \approx 2R_{PR}$
- **Depletion** (factor 10-100) of $R \approx 100R_{PR}$ bodies (\sim sub-mm)

- importance of cratering impacts
- dominant role of « foreign-born » grains coming from the inner parts

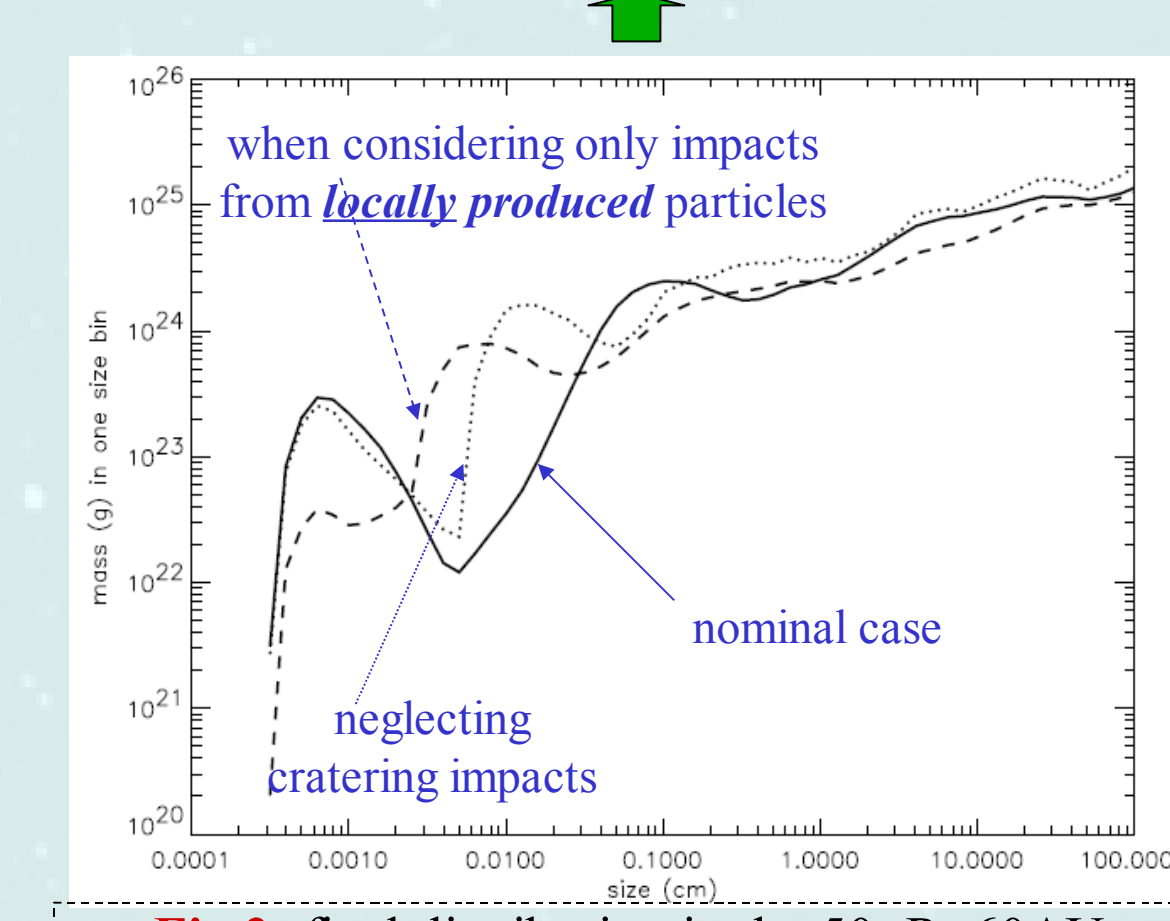
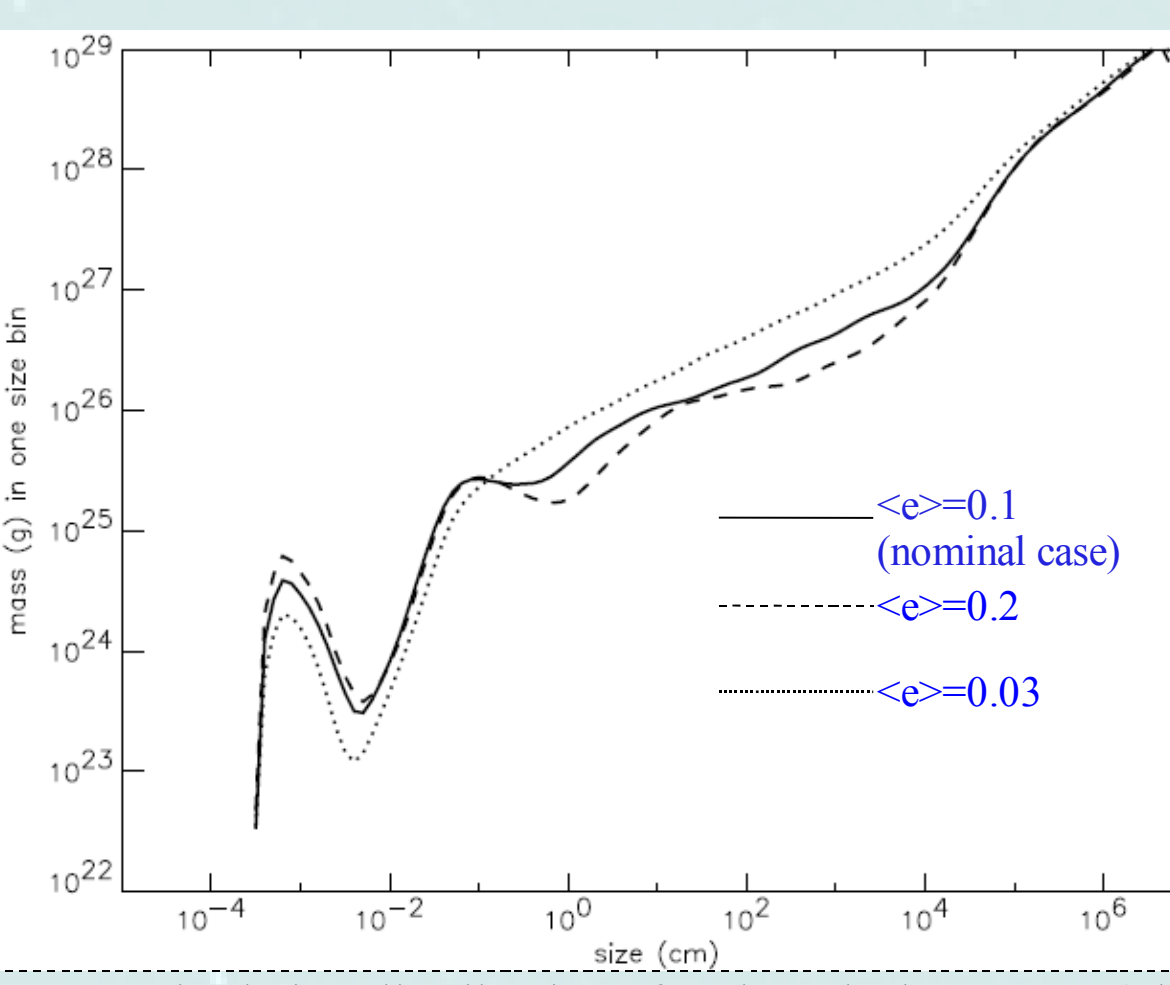


Fig 2: final distribution in the 50-60AU annulus, when neglecting specific types of impacts



Wavy structure only weakly depends on $\langle e \rangle$, because it is mostly imposed by small high- β grains

IV observable counterparts (work in progress...)

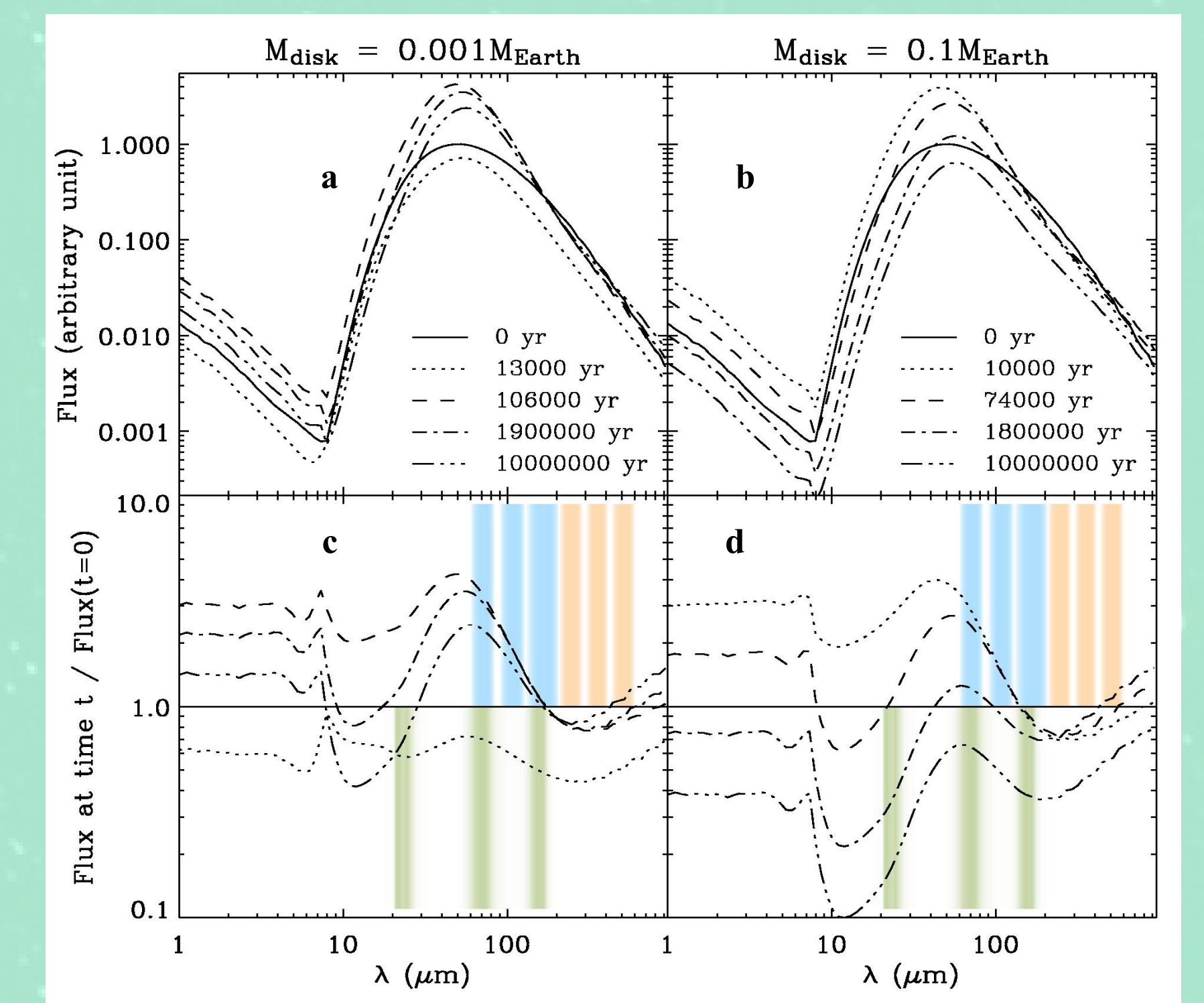


Fig 7, a,b: Synthetic SEDs, computed from the successive size distributions in the nominal case, a: high-mass ($0.1 M_{\oplus}$) and b: low-mass ($0.001 M_{\oplus}$) cases
 Fig 7, c,d: temporal evolution of the SEDs, when renormalized to the initial distribution at $t=0$ (i.e. $M.M.S.N + dN \propto R^{-3.5} dR$)
 Blue stripes: Herschel/PACS photometric lines
 Red stripes: Herschel/SPiRE photometric lines
 Green stripes: Spitzer/MIPS photometric lines

- Initial conditions**
- Nominal Case *Realistic debris disc*
- Radial extension $10 < a < 120\text{AU}$ ($11 \times 10\text{AU}$ annuli)
 - Density distribution: $\sigma = \sigma_0 a^{-1.5}$ (M.M.S.N)
 - Initial size distribution: $dN = C R^{-3.5} dR$
 - Total dust mass ($< 1\text{cm}$): $0.1 M_{\oplus}$ or $0.001 M_{\oplus}$
 - Sublimation distance for ices: 20 AU
 - Dynamical State: $\langle e \rangle = 2 \langle i \rangle = 0.1$
 - Star $M = 1.7 M_{\text{Sun}} \leftrightarrow$ blow-out size $R_{PR} = 5 \mu\text{m}$

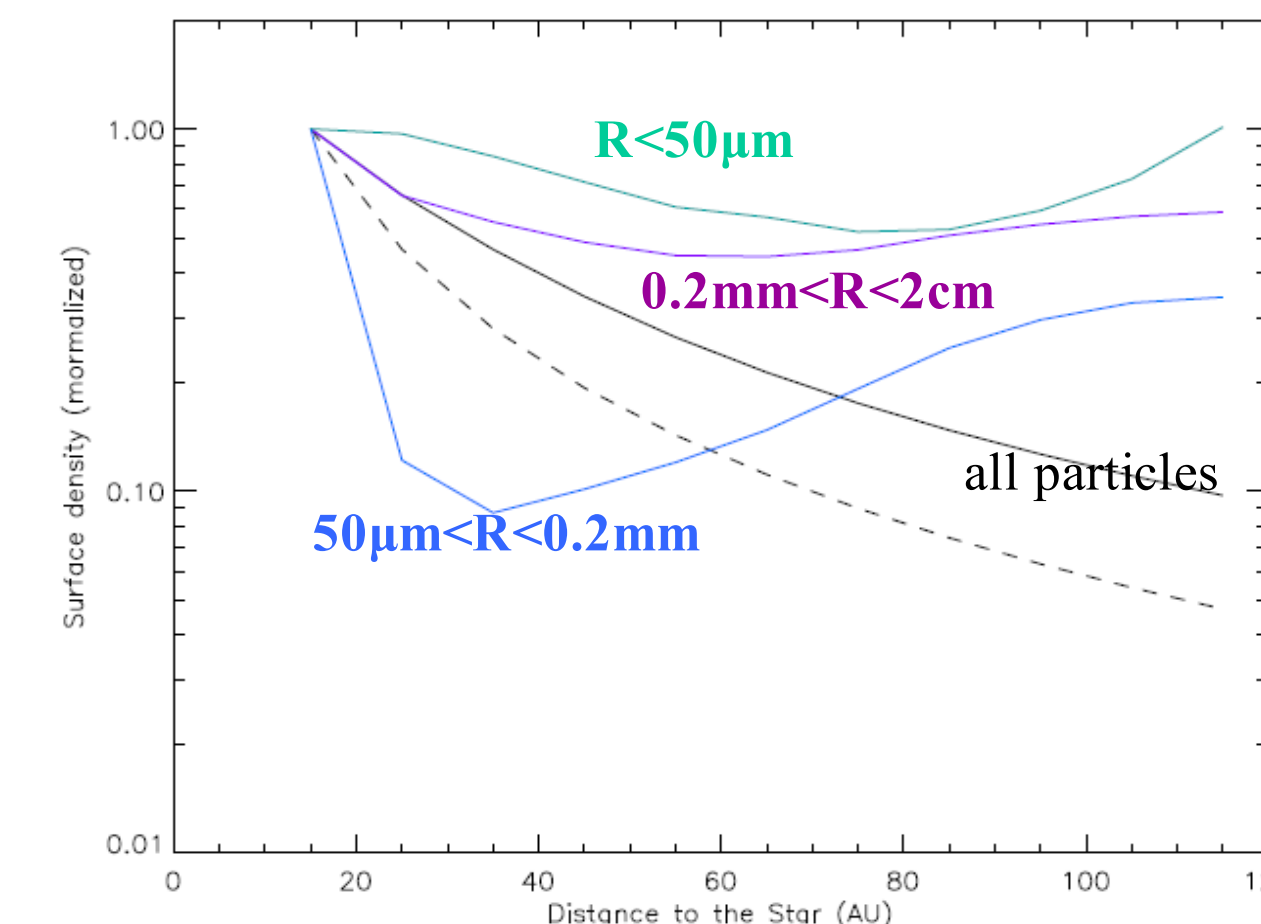


Fig 6: High-mass disc. Radial distribution, at $t = 10^7$ years, of the surface number density for different object sizes. For each size range, all surface densities are renormalized to the surface density in the first annulus. The dashed lines represent the theoretical distribution should a MMSN power law in $a^{-1.5}$ hold starting at the innermost annulus.

Departure from the MMSN in $\sigma \propto a^{-1.5}$!

bibliography

- Benz, W., Asphaug, E., 1999, Icarus, 142, 5
- Campo-Bagatin, Celino, Davis, Farinella, Paolicchi, 1994, Planet. Space Sci., 42, 1079
- Kenyon, S., Bromley, B., 2004, AJ, 127, 1
- Kenyon, S., Bromley, B., 2004, ApJ, 602, L133
- Koschny, D., Grün, E., 2001, Icarus, 174, 105
- Krivov, A., Sremecvic, Spahn, F., 2005, Icarus, 174, 105
- Krivov, A., Lohne, T., Sremecvic, M., 2006, A&A, in press
- Thébault, P., Augereau, J.C., Beust, H., 2003, A&A, 408, 775
- Thébault, P., Augereau, J.-C., 2006, to be submitted to A&A
- Wyatt, M., Dent, W., 2002, MNRAS, 334, 589

Crucial Role of the collision-outcome prescription (poorly constrained parameter!)