Eccentricity evolution of giant planet orbits due to circumstellar disk torques

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Introduction

The extrasolar planets discovered to date orbit their host stars with larger eccentricities and smaller semi-major axes than do similarly sized planets in our own solar system. It is thought that these gas giant planets could not have formed at the small radii where they are observed, but instead formed at larger radii and subsequently moved inward via planetary migration. The observed planets are massive enough to undergo Type II migration, in which the planet clears a gap in the disk and is then driven inward by resonances between the planet and material in the remainder of the disk.

The disk's effect on orbital eccentricity ε is debated, with recent analytic calculations promoting eccentricity excitation and numerical studies promoting eccentricity damping (e.g., Papaloizou et al. 2001). This project addresses this controversy using a quasi-analytic approach, removing several approximations from the calculation of disk torques. Additionally, we investigate the extent to which disk and planet properties influence eccentricity evolution, and outline a recipe for calculating $d\varepsilon/dt$. We find that:

- $d\varepsilon/dt$ takes on both positive and negative values depending on the properties of the disk and planet,
- the width and placement of the planet within the gap in disk material strongly affects $d\varepsilon/dt$, and
- the inclusion of corotation resonance saturation produces strong eccentricity excitation.

Methods and Initial Conditions

Disk Torques

Resonances occur where the ratio of the mean orbital motions of the planet and a ring of disk material is a rational number. The strongest torques affecting the orbital evolution of the planet are Lindblad and corotation torques, which correspond to rings in the disk at radii:

$$r_L = a\left(\frac{m\pm 1}{\ell}\right)^{2/3}, \qquad r_C = a\left(\frac{m}{\ell}\right)^{2/3} \qquad (1)$$

(Goldreich & Tremaine, 1980, hereafter GT80).

Parameters

M_P	$=$	$1\ M_J$
M_D	$=$	$0.05\ M_\odot$
R_D	$=$	30 AU
Σ	$=$	$\Sigma_0(r/r_0)^{-1/2}$
T	$=$	$5000\ \mathrm{K}\ (r/\ 0.014\ \mathrm{AU})^{-3/4}$
α	$=$	10^{-3}

These parameters ensure the disk is flat and non self-gravitating.

Our method can be summarized as follows:

- We choose the radial surface density profile of the disk and the orbital properties of the planet.
- We evaluate the torque and resulting contribution to $d\varepsilon/dt$ for each contributing resonance.
- We sum these contributions over all resonance types and values of *m* to determine the total $d\varepsilon/dt$.

Calculating Disk Torques

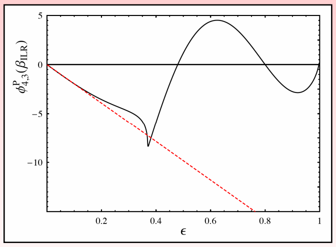
The torque functions are proportional to simple combinations of disk surface density, the slope of the surface density profile, and orbital parameters. Furthermore, the torques are a function of the cosine expansion of the planet's disturbing function, $\phi^P_{\ell,m}$:

$$T^L_{\ell,m} \propto \left(\frac{r\,d\phi^P_{\ell,m}}{dr} \mp 2m\phi^P_{\ell,m}\right)^2\Bigg|_{r_L}, \qquad T^C_{\ell,m} \propto \left(\phi^P_{\ell,m}\right)^2\Big|_{r_C} \qquad (2)$$

where

$$\phi^P(r,\theta,t) = GM_P\left(\frac{\mathbf{r}_P\cdot\mathbf{r}}{r^3} - \frac{1}{|\mathbf{r}-\mathbf{r}_P|}\right) = \sum_{\ell=-\infty}^{\infty}\sum_{m=0}^{\infty}\phi^P_{\ell,m}(r)\cos(m\theta-\ell\Omega_P t) \qquad (3)$$

(GT80). Evaluating the function $\phi^P_{\ell,m}$ is computationally expensive, and so several methods have been developed to sidestep its full calculation. One such method is expanding linearly in eccentricity. This technique is pertinent to our solar system, which contains non eccentric orbits, but is not appropriate for studying the eccentric orbits of the observed extrasolar planets. This is illustrated in the plot at right, which displays both the linear approximation (in red) and the numerically determined solution (in black) of $\phi^P_{\ell,m}$. Clearly, an approximation to first order in eccentricity is only useful for eccentricities less than 0.3.



Another method for avoiding full computation of the coefficients $\phi^P_{\ell,m}$ is to use the large *m* approximation, under which Lindblad and corotation resonances have the same functional dependence on $\phi^P_{\ell,m}$. However, we avoid this approximation also, as it is valid only for planets significantly less massive than Jupiter.

This study differs from previous works in its full numerical calculation of the coefficients of the cosine expansion of the perturbing function, $\phi^P_{\ell,m}$.

Counting Resonances

The strongest resonances occur for $\ell = m, m\pm 1$, limiting the number of values of ℓ we must consider (GT80). An infinite number of *m* resonances remain, but, as *m* increases, the location of the resonance approaches the semi-major axis of the perturbing planet (see Eq. 1). If the planet clears a gap, as in Type II migration, the edges of the gap place a physical upper limit on *m*; if, instead, the gap edges are not sharp and a small amount of gas remains, we either sum over the resonances until the result converges, or use the Roche lobe radius to provide an upper limit on *m* .

Results

With the framework developed in the above sections, we can calculate the eccentricity time derivative for a given Jovian planet in a given thin disk. The most important parameters turn out to be the width of the gap and the placement of the planet within it. All plots displayed below are for Jupiter-mass planets.

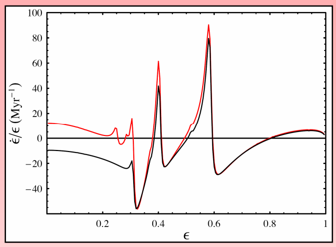


Figure 2: The simplest case: a planet orbiting at 1 AU in a sharply-defined gap. The resulting function $d\varepsilon/dt$ (black curve) exhibits neither uniform eccentricity damping nor excitation, but widely varying levels of damping or excitation depending on the current value of ε. The red curve corresponds to a slightly narrower gap, and has positive rather than negative eccentricity evolution for small ε, allowing a lone planet in a circular orbit to obtain non-zero eccentricity through its interaction with a disk.

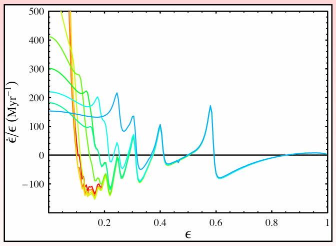


Figure 3: Here, we choose a gap shape containing some residual material. As the color of the function changes from blue to red, $d\varepsilon/dt$ contains more resonance terms, smoothed over the scale height *h*. While the low ε end of the function diverges, the rest of the plot converges, producing a "preferred" eccentricity region between 0.1 and 0.8.

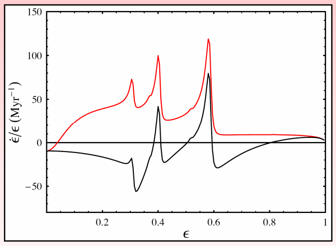
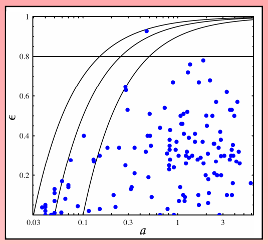


Figure 4: The result of including saturation effects (see Ogilvie & Lubow 2003) on $d\varepsilon/dt$. The red curve accounts for saturation, the black curve does not. Taken at face value, this result would imply that all planets starting with ε > 0.05 would be ejected or accreted from their systems within 0.15 Myr. Furthermore, uniform eccentricity excitation would lead to no multiple planet systems (Moorhead & Adams 2005), in contrast with the observed population of extrasolar planets which contains about 10-20% multiple systems.

Conclusions

We have shown that performing a full calculation of the expansion coefficients of the perturbing function $\phi^P_{\ell,m}$ produces an eccentricity time derivative that attains both positive and negative values.



The shape of the surface density profile near the gap has two important effects on $d\varepsilon/dt$: [1] Altering the gap width produces drastic changes in the shape of $d\varepsilon/dt$ as resonances are included or excluded. [2] Removing material from the outer edge of the gap can significantly reduce the effect of the disk on the planet. We find, regardless of gap shape, that $d\varepsilon/dt$ is always positive for ε > 0.8. This result, combined with circularization of planetary orbits with periastron less than 0.01 - 0.03 AU, well circumscribes the region of the *a*-ε plane inhabited by the extrasolar planets, as seen at right.

Finally, we find that if corotation resonances are saturated, we obtain almost exclusive eccentricity excitation. However, this excitation is so severe that a planet could only remain in orbit about its host star for about a tenth of a million years.

This work has several limitations. First, we have assumed throughout that the disk is infinitesimally thin. Second, we have assumed throughout that the disk is axisymmetric; for an eccentric planet orbit, this will not be completely true. Finally, our method for computing the level of saturation of corotation torques is an interpolation between two limits, and it may thus be important to compute the saturation level more accurately.

There are several possible extensions of this work. We have assumed a thin disk throughout this project. Thus, the first possible extension is inclusion of next order corrections for a finite disk scale height, which will affect the amplitude of second order Lindblad resonances and create an apsidal resonance, which damps ε. Finally, these results can readily be inserted into numerical studies of planet migration.

References

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