Dust diffusion in protoplanetary discs by magnetorotational turbulence

A. JOHANSEN[1], H. KLAHR[1] AND A.J. MEE[2]

[1]Max-Planck-Institut für Astronomie, 69117 Heidelberg, Germany [2]School of Mathematics and Statistics, University of Newcastle upon Tyne, UK

ABSTRACT

We measure the turbulent diffusion coefficient of dust grains embedded in magnetorotational turbulence in a protoplanetary disc directly from numerical simulations and compare it to the turbulent viscosity of the flow. The simulations are done in a local coordinate frame comoving with the gas in Keplerian rotation. Periodic boundary conditions are used in all directions, and vertical gravity is not applied to the gas. Using a two-fluid approach, small dust grains of various sizes (with friction times up to $\Omega_0\tau_f = 0.02$) are allowed to move under the influence of friction with the turbulent gas. We measure the turbulent diffusion coefficient of the dust grains by applying an external sinusoidal force field acting in the vertical direction on the dust component only. This concentrates the dust around the mid-plane of the disc, and an equilibrium distribution of the dust density is achieved when the vertical settling is counteracted by the turbulent diffusion away from the mid-plane. Comparing with analytical expressions for the equilibrium concentration we deduce the vertical turbulent diffusion coefficient. The vertical diffusion coefficient is found to be lower than the turbulent viscosity and to have an associated vertical Schmidt number of about 1.5. A similar radial force field also allows us to measure the radial turbulent diffusion coefficient. We find a radial diffusion Schmidt number of about 0.85 and also find that the radial turbulent diffusion coefficient is around 70% higher than the vertical. As most angular momentum transport happens through magnetic Maxwell stresses, both the vertical and the radial diffusion coefficients are found to be significantly higher than suggested by the angular momentum transport by Reynolds stresses alone. We also find evidence for trapping of dust grains of intermediate friction time in turbulent eddies.

1 Introduction

Turbulent gas ⇒Random dust motions ⇒Transport of dust grains⇒

Turbulent diffusion

Diffusion flux is often assumed proportional to dust-to-gas ratio gradient (Dubrulle et al., 1995),

$$\mathcal{F}_t \propto \rho \boldsymbol{\nabla} \left(\frac{n}{\rho}\right) . \tag{1}$$

Proportionality factor D_t is the turbulent diffusion coefficient. For constant gas density ρ the diffusion equation is

$$\frac{\partial n}{\partial t} = D_t \boldsymbol{\nabla}^2 n . \tag{2}$$

A very popular assumption is

$$D_t \equiv \nu_t = \alpha_t c_s^2 \Omega_0^{-1} , \tag{3}$$

where ν_t is the turbulent viscosity and α_t is the alpha-value. Definition of Schmidt number:

$$\mathrm{Sc} \equiv \frac{\nu_t}{D_t} \tag{4}$$

The Schmidt number measures the ratio of angular momentum transport to mass diffusion.

2 Questions

We wish to address the following questions:

- Is turbulent transport of dust grains well-described as diffusion?
- Is the turbulent diffusion coefficient really equal to the turbulent viscosity?
- Is the turbulent diffusion coefficient in a protoplanetetary disc isotropic?

Answer from computer simulations of magnetorotational turbulence.

For numerical solution we use the Pencil Code (6th order in space, 3rd order in time MHD code). Dust grains are treated as a fluid that interacts with the gas through a drag force.

3 Magnetorotational turbulence in a box

Equation of motion, continuity equation and induction equation for ideal MHD in the shearing sheet approximation (simulation box corotating with the disc at Keplerian angular velocity Ω_0 at a distance $r = r_0$ from the central object):

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} - 2\boldsymbol{\Omega}_0 \times \boldsymbol{u} + 3\Omega_0^2 \boldsymbol{x} - \frac{1}{\rho}\boldsymbol{\nabla} P + \frac{1}{\rho}\boldsymbol{J}\times\boldsymbol{B} \tag{5}$$

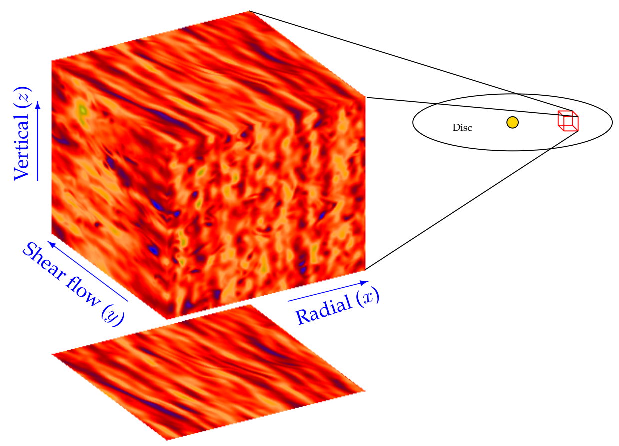
$$\frac{\partial \rho}{\partial t} = -\rho\boldsymbol{\nabla}\cdot\boldsymbol{u} - \boldsymbol{u}\cdot\boldsymbol{\nabla}\rho \tag{6}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u}\times\boldsymbol{B} \tag{7}$$

Coriolis force ($-2\boldsymbol{\Omega}_0 \times \boldsymbol{u}$); Centrifugal force and radial gravity ($3\Omega_0^2 \boldsymbol{x}$)

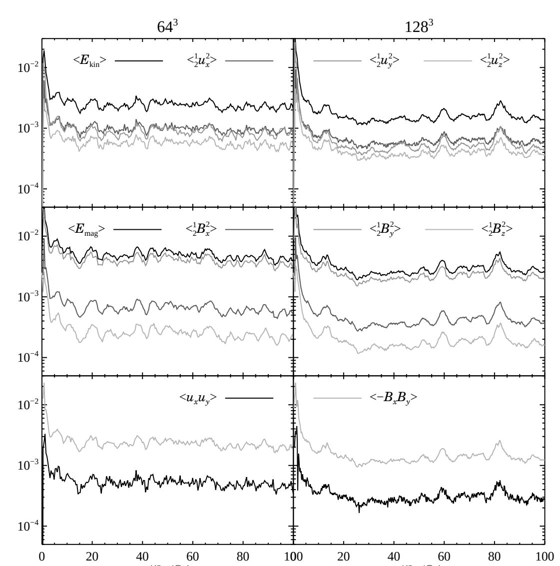
There is no vertical gravity on the gas. Periodic boundary conditions are applied in all directions. The initial condition is random perturbations in gas velocity and a sinusoidal perturbation in magnetic vector potential. This is unstable due to magnetorotational instability (Balbus & Hawley, 1991; Brandenburg et al., 1995).

Density contours at the sides of the simulation box after 6.5 orbits:



4 Evolution of MRI turbulence

Kinetic energy (first row), magnetic energy (second row) and Reynolds and Maxwell stresses (third row) for resolutions of 64^3 and 128^3:



The turbulence has an α-value of approximately $\alpha = 0.002$. This is for a simulation with no net magnetic field through the box. When imposing a vertical field, the turbulence becomes much stronger (see Section 7).

5 Dust

Equation of motion and continuity equation (including drag force and external gravity):

$$\frac{\partial \boldsymbol{w}}{\partial t} = -(\boldsymbol{w}\cdot\boldsymbol{\nabla})\boldsymbol{w} - 2\boldsymbol{\Omega}_0 \times \boldsymbol{w} + 3\Omega_0^2 \boldsymbol{x} - \frac{1}{\tau_f}(\boldsymbol{w}-\boldsymbol{u}) + \boldsymbol{g} \tag{8}$$

$$\frac{\partial n}{\partial t} = -n\boldsymbol{\nabla}\cdot\boldsymbol{w} - \boldsymbol{w}\cdot\boldsymbol{\nabla} n \tag{9}$$

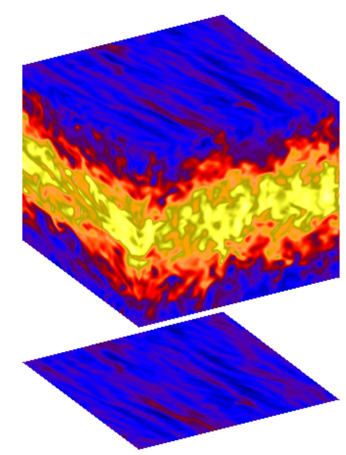
We apply an external sinusoidal gravity $g_z = g_0 \sin(k_z z)$. This leads to non-zero equilibrium dust density distribution where settling due to vertical gravity is balanced by turbulent transport of dust grains away from the mid-plane. We expect the analytical equilibrium solution

$$\ln n(z) = B + A\cos(k_z z) , \tag{10}$$

where the amplitude of the cosine is

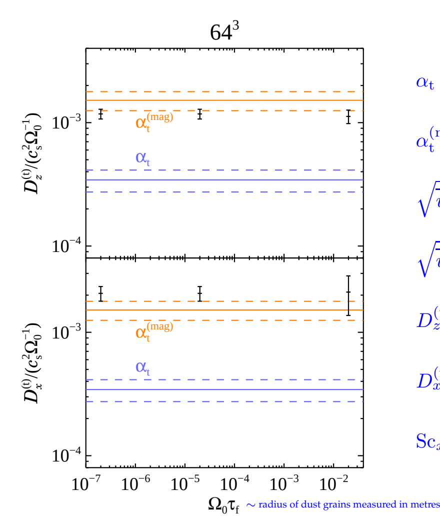
$$A = \frac{\tau_f g_0}{k_z D_t} . \tag{11}$$

Dust number density contours at the sides of the box after 6.5 orbits:



Best-fit amplitude A gives value of $D_z^{(t)}$. Same can be done with a radial gravity to yield $D_x^{(t)}$.

6 Measured diffusion coefficients



$\alpha_t = 3.4 \cdot 10^{-4}$

$\alpha_t^{(mag)} = 1.5 \cdot 10^{-3}$

$\sqrt{\overline{u_z^2}} = 0.033 c_s$

$\sqrt{\overline{u_x^2}} = 0.044 c_s$

$D_z^{(t)} = 1.1 \cdot 10^{-3} c_s^2 \Omega_0^{-1}$

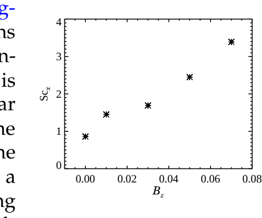
$D_x^{(t)} = 2.1 \cdot 10^{-3} c_s^2 \Omega_0^{-1}$

$\mathrm{Sc}_x = 0.85$; $\mathrm{Sc}_z = 1.5$

Diffusion coefficient is anisotropic. The radial diffusion is stronger than the turbulent viscosity ($\mathrm{Sc}_x = 0.85$), while the vertical diffusion is weaker ($\mathrm{Sc}_z = 1.5$). As most angular momentum transport happens through magnetic Maxwell stresses, both the vertical and the radial diffusion coefficients are found to be significantly higher than suggested by the angular momentum transport by Reynolds stresses alone.

7 Dependence on net vertical field

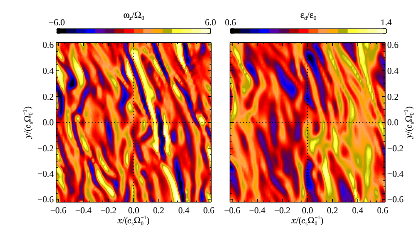
The strength of the MRI turbulence depends on the size of an externally imposed magnetic field B_z. The presented simulations were all done with $B_z = 0.0$. The dependence of the radial Schmidt number on B_z is shown on the right. There is an almost linear rise of Sc_x with B_z. This may explain some of the difference between our work and the work by Carballido et al. (2005), who find a radial Schmidt number of 10 when applying a net vertical field of $B_z = 0.07$ (corresponding to a plasma beta value of $\beta = 400$).



8 Dust-trapping in turbulent eddies?

For $\Omega_0\tau_f \sim 0.01$ the diffusion coefficient fluctuates a lot. Could be due to trapping of dust grains in turbulent eddies (Barge & Sommeria, 1995; Chavanis, 2000; Johansen et al., 2004). If so, expect a correlation between vertical vorticity and dust-to-gas ratio. Vorticity is defined as $\boldsymbol{\omega} \equiv \boldsymbol{\nabla}\times\boldsymbol{u}$.

Plot shows contours of minus vertical vorticity (left) and dust-to-gas ration (right):



There is a clear correlation between blue regions on left plot and yellow regions regions on right plot (and vice versa). Dust grains are trapped in regions of negative vertical vorticity (anticyclones) and expelled from regions of positive vertical vorticity (cyclones).

9 Conclusions

- The radial diffusion is twice as strong as the vertical diffusion. Both are comparable in size to the total turbulent viscosity. This is surprising, since the larger part of the turbulent viscosity is purely magnetic.
- There is no apparent dependence of D_t on friction time (but the grains considered are all small, so this is expected).
- There is a tendency for trapping dust grains in regions of negative vertical vorticity (anticyclones) and expelling from regions of positive vertical vorticity (cyclones).

This work:
Johansen & Klahr (2005), Johansen, Klahr & Mee (2006)

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